A Stochastic Integral Equation Method for Modeling the Rough Surface Effect On Interconnect Capacitance *

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Abstract

In this paper we describe a stochastic integral equation method for computing the mean value and the variance of capacitance of interconnects with random surface roughness. An ensemble average Green's function is combined with a matrix Neumann expansion to compute nominal capacitance and its variance. This method avoids the time-consuming Monte Carlo simulations and the discretization of rough surfaces. Numerical experiments show that the results of the new method agree very well with Monte Carlo simulation results.

1. Introduction

Many of the fabrication processes used to generate both on- and off-chip interconnect will produce conductors with surface variations. Though these variations may not be truly random, they can often be accurately modeled by assuming random surface roughness with an appropriate spatial correlation. Experiments indicate that interconnect can easily have topological variations with peak to valley distances larger than five microns [1, 2]. Measurements indicate that this surface roughness can increase high frequency resistance by as much as a factor of three[1], and analytical treatments of the surface roughness problem [3, 4] correlates well with these measurements. It has also been show that capacitance is significantly increased by the surface roughness [5, 6].

Though 3D parasitic extraction has improved substantially recently [7, 8, 9, 10, 11, 12, 13], these programs were developed to analyze 3D structures with smooth surfaces. To the best of our knowledge, there has been little work on numerical techniques specifically designed for analyzing three dimensional interconnect with rough surfaces. Although it is possible to use existing programs to analyze conductors with rough surfaces, such approaches are slow for two reasons. First, the details in the random profile of rough surfaces requires a very fine discretization, and second, an ensemble of analyses must be performed to estimate the mean and variance of the extracted parameters.

The effect of surface roughness in the context of electromagnetic radiation and scattering has been studied for at least three decades [14]. Work on the analysis of rough surface effect falls roughly into two broad categories: numerical simulation techniques [3, 15] and approximate analytical techniques [5, 16]. In the numerical simulation approach, the statistical nature of the rough surface model is commonly dealt with using computationally intensive Monte Carlo methods [15]. A surface impedance boundary condition was proposed in [4] to take into account the roughness effect. This strategy avoids the discretization of rough surfaces. But only 2D grooves with periodic roughness are analyzed in [4]. More importantly, since the impedance boundary condition depends on the profile of the 2D groove, this approach does not avoid the time-consuming Monte Carlo simulations. On the other hand, since the solutions of the approximate analytical approach are explicit analytical forms, it is possible to calculate the mean value and even the variance directly [16]. However, many assumptions have to be made in the approximate analytical techniques, hence limiting its application. A further step was made in [17], where the mean scattering field in a 2D rough surface acoustic scattering problem was calculated directly without using the Monte Carlo process. In this case, the analytical solution is not readily available. The ensemble average was taken on both sides of the governing integral equation instead of on the analytical solution as in [16]. This leads to an integral equation of the mean scattering field defined only on the smooth surface with the surface roughness removed. Since only a 2D rough surface is considered in [17], it is possible to use analytical techniques such as the Laplace transformation to obtain the solution to the mean field integral equation.

In this paper, we extend the ensemble average Green's function idea in [17] to the numerical solution of stochastic integral equations. A crucial assumption, the uncorrelatedness between source and Green's function, is used in [17] to significantly simplify the formulation. The justification for this assumption in [17] is based on physical intuition. In this paper, we first demonstrate a mathematical interpretation of this assumption and show that it leads to inaccurate results when the surface roughness magnitude is large. We then propose a correction scheme to substantially improve the accuracy. In addition, we have extended the ensemble average Green's function idea to the calculation of the variance. Finally, we demonstrate the method on some relatively simple capacitance problems to show that it is possible to directly calculate the mean surface charge density, the mean and the variance of capacitance by just one solve, as oppose to many thousands of solves in Monte Carlo approach.

2. Mathematical Model for Rough Surfaces

The rough surface of a conductor can be described as a statistical perturbation of a nominal smooth surface. Specifically, for each point $\vec{r}$ on the smooth surface, let $h(\vec{r})$ be a perturbation normal to the smooth surface at point $\vec{r}$. For typical rough surfaces, $h(\vec{r})$ is described by a probability density function with spatial correlation. A common such model is the Gaussian distribution

$$P_1(h) = \frac{1}{\sqrt{2\pi}\sigma} exp(-\frac{h^2}{2\sigma^2}) \tag{1}$$

for each individual point and the Gaussian joint distribution

$$P_2(h_1,h_2;\vec{r}_1,\vec{r}_2)) = \frac{exp(-\frac{h_1^2-2C(\vec{r}_1,\vec{r}_2)h_1h_2+h_2^2}{2\sigma^2(1-C(\vec{r}_1,\vec{r}_2)^2)})}{2\pi\sigma^2\sqrt{1-C(\vec{r}_1,\vec{r}_2)^2}} \tag{2}$$

for the connection between two points on the same surface. Here σ is the standard deviation and $C(\vec{r}_1,\vec{r}_2)$ is the auto-correlation function. We assumed that the random rough surface is translation invariant, i.e.,

$$C(\vec{r}_1,\vec{r}_2) = C(|\vec{r}_1-\vec{r}_2|) = C(\xi). \tag{3}$$

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The most commonly used auto-correlation function is the Gaussian correlation function

$$C(\xi) = exp(-\frac{\xi^2}{\eta^2}), \tag{4}$$

where η is correlation length, defined as the value of ξ at which $C(\eta) = \frac{1}{e}$ where e is Euler constant.

The height fluctuation defined by (1), (2) and (3) is a stationary Gaussian stochastic process [18]. From above description it is clear that this stochastic process is uniquely determined by two parameters: the standard deviation σ and the correlation length η. Here we want to emphasize that the stochastic integral equation (SIE) method developed in this paper is not tied to the specific mathematical model for rough surface. A different model for rough surface simply means that different probability density functions like the ones in (1) and (2) will be used in calculating the ensemble average Green's function. But the basic elements of the SIE method remain the same.

3. Stochastic Integral Equation Method

For the sake of clarity, we use a simple 2D capacitance problem, a single conductor over a ground plane, to explain the basic ideas of the stochastic integral equation method. As will be made clear, this method can be readily extended to the multiple conductor case as well as to the 3D capacitance problems with little modification.

Figure 1 shows one conductor over an infinite ground plane, where the conductor is denoted as D_1 and the ground plane is denoted as D_0. Without loss of generality, we assume that the side walls of conductor D_1 (denoted as S_1 and S_3) are smooth, only the top and the bottom surfaces (denoted as S_2 and S_4) are rough. The position of the points on the top and the bottom surfaces are defined by

$$\begin{cases} y_1(x) = b + h_1(x) & (x, y_1(x)) \in S_2 \\ y_2(x) = a + h_2(x) & (x, y_2(x)) \in S_4 \end{cases} \tag{5}$$

where $h_1(x)$ and $h_1(x)$ are two independent surface height fluctuation functions with statistical characteristics defined by (1), (2) and (3), and a and b are the nominal position of the top and the bottom surface, respectively, as shown in figure 1. To facilitate the explanation in the following sections, we also define the smooth nominal surfaces $\tilde{S_2}$ and $\tilde{S_4}$ for rough surfaces S_2 and S_4 as $\{(x,y) \in \tilde{S_2} | 0 \le x \le c, y = b\}$ and $\{(x,y) \in \tilde{S_4} | 0 \le x \le c, y = a\}$, respectively.

3.1 Description of 2D capacitance problem

The 2D capacitance can be calculated by solving a 2D exterior Laplace problem defined as

$$\begin{cases} (\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2})\phi(x,y) = 0 \ \ (x,y) \in D \\ \phi(x,y) = V_i \quad (x,y) \in \partial D_i \quad i = 0,1 \\ \phi(x,y) = 0 \quad (x,y) \to \infty \end{cases} \tag{6}$$

where D denotes the region outside of conductors, ∂D_i refers to the surface of D_i and V_i is the given voltage on ∂D_i. To compute capacitance, we set $V_1 = 1$ and $V_0 = 0$. Equation (6) can be converted to the equivalent integral equation [7]

$$\int_{\partial D_1} dl(x',y') \frac{\rho(x',y')}{\varepsilon_0} G(x',y';x,y) = V_1 = 1, \quad (x,y) \in \partial D_1, \tag{7}$$

where

$$G(x',y';x,y) = \frac{1}{2\pi} \ln \sqrt{\frac{(x-x'')^2 + (y-y'')^2}{(x-x')^2 + (y-y')^2}}, \tag{8}$$

and (x'', y'') is the image of (x', y') with respect to the ground plane. Here we have used the image theory ([19], pp. 48) to take into account the effect of ground plane. So in our calculation, we only need to discretize conductor surface ∂D_1. It should be noted that with the Green's function in (8) the boundary conditions at infinity and on the ground plane are satisfied automatically. Using the fact that the rough surface height is a function of position, and combining that with the standard change of variable identity for integrals yields

$$\begin{aligned} \int_a^b \frac{\rho(x'=0,y')}{\varepsilon_0} G(x'=0,y';x,y) dy' \quad + \\ \int_0^c \sqrt{1+(\frac{dy_1(x')}{dx'})^2} \frac{\rho(x',y_1(x'))}{\varepsilon_0} G(x',y_1(x');x,y) dx' \quad + \\ \int_a^b \frac{\rho(x'=c,y')}{\varepsilon_0} G(x'=c,y';x,y) dy' \quad + \\ \int_0^c \sqrt{1+(\frac{dy_2(x')}{dx'})^2} \frac{\rho(x',y_2(x'))}{\varepsilon_0} G(x',y_2(x');x,y) dx' \quad = \quad 1 \\ (x,y) \in \partial D_1, \end{aligned} \tag{9}$$

where the first and the third terms are associated with the two smooth sides and the second and the fourth terms are associated with the rough top and bottom (see figure 1). Now define

$$\tilde{\rho}(x',y') = \begin{cases} \rho(x',y'), \quad (x',y') \in S_1, S_3 \\ \sqrt{1+(\frac{dy_1(x')}{dx'})^2} \rho(x', y_1(x')), \quad (x',y') \in \tilde{S}_2 \\ \sqrt{1+(\frac{dy_2(x')}{dx'})^2} \rho(x', y_2(x')), \quad (x',y') \in \tilde{S}_4, \end{cases} \tag{10}$$

$$d\tilde{l}(x',y') = \begin{cases} dx', \quad (x',y') \in \tilde{S}_2, \tilde{S}_4 \\ dy', \quad (x',y') \in S_1, S_3, \end{cases} \tag{11}$$

$$\hat{G}(x',y';x,y) = \begin{cases} G(x',y';x,y), \quad (x',y') \in S_1, S_3 \\ G(x',y_1(x');x,y), \quad (x',y') \in \tilde{S}_2 \\ G(x',y_2(x');x,y), \quad (x',y') \in \tilde{S}_4, \end{cases} \tag{12}$$

then equation (9) can be written as

$$\int_{\partial \tilde{D}_1} \frac{\tilde{\rho}(x',y')}{\varepsilon_0} \hat{G}(x',y';x,y) d\tilde{l}(x',y') = 1, \quad (x,y) \in \partial D_1, \tag{13}$$

where $\partial \tilde{D}_1$ is the nominal smooth surface. It should be pointed out that with the change of variable defined in (10) and (11), the unknown charge $\tilde{\rho}$ is define on the nominal smooth surface $\partial \tilde{D}_1$ and the integral domain of equation (13) becomes $\partial \tilde{D}_1$. This makes it much easier to use the standard definition of stochastic integral [18] in the following sections. [1] In view of (10) and (11), the self capacitance is

$$C = \int_{\partial D_1} \rho(x',y') dl(x',y') = \int_{\partial \tilde{D}_1} \tilde{\rho}(x',y') d\tilde{l}(x',y'). \tag{14}$$

It should be noted that the charge density distribution is a nonlinear function of the surface point location, as shown in (7). This implies that the capacitance is also a nonlinear function of the surface point location. Hence the mean capacitance is not equal to the capacitance for the conductor with a nominal smooth surface. This should not be surprising because the rough surface conductor's surface area is larger. As shown in [6] as well as in our numerical result section, this difference is not negligible.

3.2 Basic ideas

Instead of solving equation (13) for many statistically independent realizations of the rough surface and taking the ensemble average of charge density, we derive the integral equation for the mean charge density directly. By solving this stochastic integral equation, we can easily calculate the mean capacitance.

Taking the ensemble average on both sides of equation (13) yields

$$\int_{\partial \tilde{D}_1} d\tilde{l}(x',y') < \frac{\tilde{\rho}(x',y')}{\varepsilon_0} \tilde{G}(x',y';x,y) >= 1, \quad (x,y) \in \partial \tilde{D}_1 \tag{15}$$

[1] The stochastic processes $y_1(x')$ and $y_2(x')$ in (10) are differentiable because the autocorrelation of $h_1(x)$ and $h_2(x)$ in (5), which is defined in (4), has derivative of order up to at least two [18].

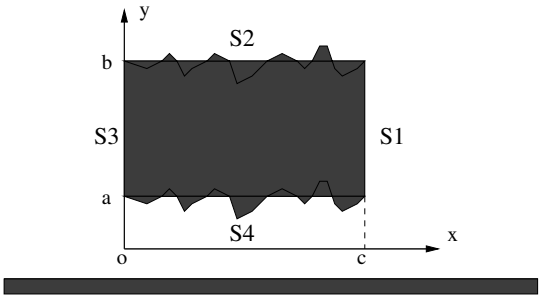

Figure 1: One conductor over a ground plane. The top and the bottom surfaces are rough with nominal position at $y = b$ and $y = a$, respectively.

where

$$\tilde{G}(x',y';x,y) = \begin{cases} \hat{G}(x',y';x,y), & (x,y) \in S_1, S_3 \\ \hat{G}(x',y';x,y_1(x)), & (x,y) \in \tilde{S}_2 \\ \hat{G}(x',y';x,y_2(x)), & (x,y) \in \tilde{S}_4, \end{cases} \tag{16}$$

and the angle brackets stand for ensemble average. Assuming that the charge density distribution is uncorrelated to Green's function, as is done in [17], equation (15) becomes

$$\int_{\partial\tilde{D}_1} d\tilde{l}(x',y') \frac{<\tilde{\rho}(x',y')>}{\varepsilon_0} <\tilde{G}(x',y';x,y)>= 1, \quad (x,y) \in \partial\tilde{D}_1. \tag{17}$$

In section 3.3 we will explain the significance of this uncorrelatedness assumption and show that there is a way to compensate for the error introduced by this approximation. In view of (14) the mean self capacitance is

$$<C>= \int_{\partial\tilde{D}_1} d\tilde{l}(x',y') <\tilde{\rho}(x',y')> . \tag{18}$$

It is clear that $<\tilde{\rho}(x',y')>$ is exactly what we want to compute, and it is treated as the unknown variable.

It should be noted that the surface roughness is not explicit in the ensemble average Green's function, hence only the smooth nominal surface $\partial\tilde{D}_1$ needs be discretized and only one solve is needed to obtain the mean charge density.

In this paper we use piecewise constant basis functions and the Galerkin method to discretize equation (17). The discretized system for (17) is

$$[\bar{A}] <\tilde{\rho}>= \tilde{L}, \tag{19}$$

where

$$\bar{A}_{k,j} = \int_{\tilde{\Delta}_k} d\tilde{l}(x,y) \int_{\tilde{\Delta}_j} d\tilde{l}(x',y') <\tilde{G}(x',y';x,y)>, \tag{20}$$

$$\tilde{L}_k = \int_{\tilde{\Delta}_k} d\tilde{l}(x,y). \tag{21}$$

The detailed formulas for evaluating (20) are given in [20].

3.3 Second order correction to the uncorrelatedness assumption

In this section we show that the solution of (19) is only a zero-th order approximation to the correct mean charge density. Hence we will call it $<\tilde{\rho}^{(0)}>$ in the remaining part of the paper and we have

$$<\tilde{\rho}^{(0)}>= \bar{A}^{-1}\tilde{L}. \tag{22}$$

The discretization of (13) on each realization of the rough surface results in

$$[A]\tilde{\rho} = \tilde{L}, \tag{23}$$

or equivalently,

$$A\tilde{\rho} = (\bar{A}+A-\bar{A})\tilde{\rho} = \bar{A}[I+\bar{A}^{-1}(A-\bar{A})]\tilde{\rho} = \tilde{L}. \tag{24}$$

Therefore,

$$\tilde{\rho} = [I+\bar{A}^{-1}(A-\bar{A})]^{-1}\bar{A}^{-1}\tilde{L}. \tag{25}$$

Using an idea from stochastic finite elements [21], Taylor expanding (25) and using (22) yield

$$\begin{aligned} \tilde{\rho} &\simeq [I-\bar{A}^{-1}(A-\bar{A})+\bar{A}^{-1}(A-\bar{A})\bar{A}^{-1}(A-\bar{A})] <\tilde{\rho}^{(0)}> \\ &= \tilde{\rho}^{(0)}+\tilde{\rho}^{(1)}+\tilde{\rho}^{(2)}, \end{aligned} \tag{26}$$

where $\tilde{\rho}^{(0)} =<\tilde{\rho}^{(0)}>$, $\tilde{\rho}^{(1)} = -\bar{A}^{-1}(A-\bar{A}) <\tilde{\rho}^{(0)}>$, and $\tilde{\rho}^{(2)} = \bar{A}^{-1}(A-\bar{A})\bar{A}^{-1}(A-\bar{A}) <\tilde{\rho}^{(0)}>$. The approximation in (26) is due to the truncated Taylor expansion. Taking the ensemble average on both side of (26) yields

$$<\tilde{\rho}> \simeq <\tilde{\rho}^{(0)}>+<\tilde{\rho}^{(2)}>, \tag{27}$$

where the elimination of the term $<\tilde{\rho}^{(1)}>$ is due to

$$<\tilde{\rho}^{(1)}>= -\bar{A}^{-1} <(A-\bar{A})><\tilde{\rho}^{(0)}>= 0. \tag{28}$$

Now it is clear that the uncorrelatedness assumption in section 3.2 only gives us the zero-th order term $<\tilde{\rho}^{(0)}>$ in (27). Hence its accuracy depends largely on the size of the deviation of each matrix A from $\bar{A}$. In other words, it depends on the magnitude of the surface roughness. In the numerical result section we show that this is indeed the case and that the second order correction term improves the accuracy significantly.

The difficulty in (27) is that in order to obtain the correction term $<\tilde{\rho}^{(2)}>$ it is necessary to compute $<(A-\bar{A})\bar{A}^{-1}(A-\bar{A})>$. In the following, we will use the Kronecker product to show how to compute this term.

As explained in section 3.1 and 3.2, both $<\tilde{\rho}>$ and $<\tilde{\rho}^{(0)}>$ are defined on the smooth nominal surface. From (27), the average capacitance is

$$\begin{aligned} <C> &= \tilde{L}^T <\tilde{\rho}> \simeq \tilde{L}^T(<\tilde{\rho}^{(0)}>+<\tilde{\rho}^{(2)}>) \\ &= <C^{(0)}>+<C^{(2)}>, \end{aligned} \tag{29}$$

where

$$\begin{aligned} <C^{(2)}> &= \tilde{L}^T <\tilde{\rho}^{(2)}> \\ &= \tilde{L}^T\bar{A}^{-1} <(A-\bar{A})\bar{A}^{-1}(A-\bar{A})> \bar{A}^{-1}\tilde{L} \\ &= <\tilde{\rho}^{(0)}>^T <E\bar{A}^{-1}E><\tilde{\rho}^{(0)}> \end{aligned} \tag{30}$$

and

$$E = A-\bar{A}. \tag{31}$$

The last equality in (30) is due to $\bar{A}^T = \bar{A}$, because the Galerkin method is used in (19). Using Kronecker product identities and the so-called *vec* operator [22, 23], (30) can be written as

$$\begin{aligned} <C^{(2)}> &\simeq <(<\tilde{\rho}^{(0)}>^T E)\bar{A}^{-1}(E<\tilde{\rho}^{(0)}>)> \\ &= <(<\tilde{\rho}^{(0)}>^T E^T)\otimes(<\tilde{\rho}^{(0)}>^T E)> vec(\bar{A}^{-1}) \\ &= (<\tilde{\rho}^{(0)}>^T \otimes <\tilde{\rho}^{(0)}>^T) <E^T\otimes E> vec(\bar{A}^{-1}) \\ &= (vec(B))^T vec(\bar{A}^{-1}) = trace(\bar{A}^{-T}B). \end{aligned} \tag{32}$$

where

$$\begin{aligned} vec(B) &= [(<\tilde{\rho}^{(0)}>^T \otimes <\tilde{\rho}^{(0)}>^T) <E^T\otimes E>]^T \\ &= <E\otimes E^T>(<\tilde{\rho}^{(0)}>\otimes<\tilde{\rho}^{(0)}>) \\ &= [F](<\tilde{\rho}^{(0)}>\otimes<\tilde{\rho}^{(0)}>) \end{aligned} \tag{33}$$

$$F = [F^{ij}]_{N\times N} = [<E_{ij}E^T>]_{N\times N} \tag{34}$$

$$F_{nm}^{ij} = < E_{ij}E_{mn} > = < (A_{ij} - \bar{A}_{ij})(A_{mn} - \bar{A}_{mn}) > = < A_{ij}A_{mn} > - \bar{A}_{ij}\bar{A}_{mn}. \quad (35)$$

3.4 Variance of the capacitance

The algorithm in previous sections yields the mean capacitance. In this section we show that the capacitance variance can be calculated by using the same Taylor expansion in (26). From (14) and (19),

$$< C^2 > = \int_{\partial\tilde{D}_1}\int_{\partial\tilde{D}_1} < \tilde{\rho}(x,y)\tilde{\rho}(x',y') > d\tilde{l}(x,y)d\tilde{l}(x',y'). \simeq \tilde{L}^T < \tilde{\rho}\tilde{\rho}^T > \tilde{L}, \quad (36)$$

Using the second-order approximation in (26) yields

$$\begin{aligned} < C^2 > &\simeq \tilde{L}^T \sum_{i=0}^{2}\sum_{j=0}^{2} < \tilde{\rho}^{(i)}(\tilde{\rho}^{(j)})^T > \tilde{L} \\ &\simeq \tilde{L}^T < \tilde{\rho}^{(0)} > (< \tilde{\rho}^{(0)} >)^T + < \tilde{\rho}^{(1)} >^T + < \tilde{\rho}^{(2)} >^T)\tilde{L} \\ &\quad + \tilde{L}^T(< \tilde{\rho}^{(1)} >< \tilde{\rho}^{(0)} >^T + < \tilde{\rho}^{(1)}(\tilde{\rho}^{(1)})^T >)\tilde{L} \\ &\quad + \tilde{L}^T < \tilde{\rho}^{(2)} >< \tilde{\rho}^{(0)} >^T \tilde{L} \\ &= < C^{(0)} >< C > + \tilde{L}^T < \tilde{\rho}^{(1)}(\tilde{\rho}^{(1)})^T > \tilde{L} \\ &\quad + < C^{(2)} >< C^{(0)} >, \end{aligned} \quad (37)$$

where the high-order terms $< \tilde{\rho}^{(1)}(\tilde{\rho}^{(2)})^T >$ and $< \tilde{\rho}^{(2)}(\tilde{\rho}^{(2)})^T >$ are truncated and the term $< \tilde{\rho}^{(1)} >< (\tilde{\rho}^{(0)})^T >$ is eliminated using (28). From (29),

$$< C >^2 \simeq < C^{(0)} >< C > + < C^{(2)} >< C^{(0)} >, \quad (38)$$

where the high-order term $< C^{(2)} >< C^{(2)} >$ is truncated. Therefore, the variance of the capacitance is

$$Var\{C\} = < C^2 > - < C >^2 \simeq \tilde{L}^T < \tilde{\rho}^{(1)}(\tilde{\rho}^{(1)})^T > \tilde{L}. \quad (39)$$

From (22), (26) and (31), using the Kronecker product identities yields

$$\begin{aligned} &\tilde{L}^T < \tilde{\rho}^{(1)}(\tilde{\rho}^{(1)})^T > \tilde{L} \\ &= < (< \tilde{\rho}^{(0)} >^T E < \tilde{\rho}^{(0)} >)(< \tilde{\rho}^{(0)} >^T E^T < \tilde{\rho}^{(0)} >) > \\ &= < (< \tilde{\rho}^{(0)} >^T E < \tilde{\rho}^{(0)} >) \otimes (< \tilde{\rho}^{(0)} >^T E^T < \tilde{\rho}^{(0)} >) > \\ &= (< \tilde{\rho}^{(0)} >^T \otimes < \tilde{\rho}^{(0)} >^T) < E \otimes E^T > (< \tilde{\rho}^{(0)} > \otimes < \tilde{\rho}^{(0)} >) \\ &= (vec(B))^T (< \tilde{\rho}^{(0)} > \otimes < \tilde{\rho}^{(0)} >) \\ &= (< \tilde{\rho}^{(0)} > \otimes < \tilde{\rho}^{(0)} >)^T vec(B) \\ &= (< \tilde{\rho}^{(0)} >^T \otimes < \tilde{\rho}^{(0)} >^T) vec(B) \\ &= < \tilde{\rho}^{(0)} >^T B < \tilde{\rho}^{(0)} > \end{aligned} \quad (40)$$

where the second equality is due to the fact that $< \tilde{\rho}^{(0)} >^T E < \tilde{\rho}^{(0)} >$ is a scalar. Both the first and the fourth equality use the fact that $\bar{A}^T = \bar{A}$ and $E^T = E$. Similar to the calculation of $< C^{(2)} >$ in (32), everything boils down to the calculation of matrix B or $vec(B)$. Therefore, no extra computation work is necessary to obtain the approximate value of the capacitance variance once we have used the second-order correction scheme in section 3.3 to calculate the mean capacitance.

4. Numerical Results

In this section, we first use a simple 2D example to verify the second-order correction scheme in section 3.3 and the method for calculating capacitance variance in section 3.4. We then use a simple 3D example to demonstrate that the stochastic integral equation method can be easily extended to 3D cases.

4.1 A two-dimensional example

The two-dimensional example is a single circular conductor over ground plane. The mean radius of the wire is $1mm$ and the radius fluctuation is a stochastic process with respect to the angle in the polar coordinate system. The surface of the ground is assumed to be smooth. The distance between the circular wire with nominal smooth surface and the ground is $0.5mm$.

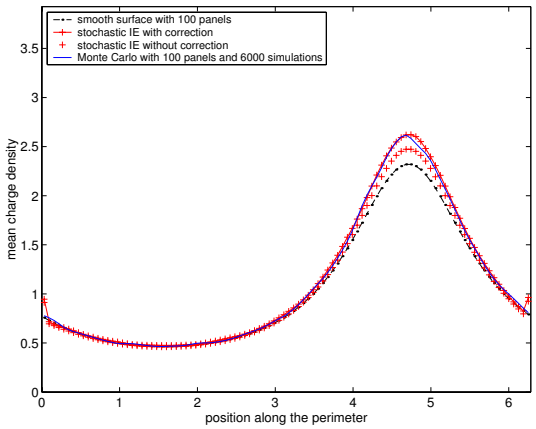

Figure 2: The mean charge density computed with Monte Carlo simulations and the stochastic integral equation method. The correlation length is $0.2mm$ and standard deviation is $0.1mm$. Maximum charge density is around the surface point where the circular wire is closest to the ground.

The mean and the variance of capacitances calculated using different methods are compared in table 1 and 2, respectively. Column *Smooth* is the capacitance for the conductor with nominal smooth surface. This serves as a reference. Column *Monte Carlo* is the capacitance computed using Monte Carlo simulations. The number of Monte Carlo runs is shown in the parenthesis. Columns *SIE I* and *SIE II* are the capacitances computed using the stochastic integral equation method without and with the second-order correction term, respectively. The parameters η (the correlation length) and σ (the standard deviation) are two numbers we use to control the roughness of the surface. Smaller η or larger σ means a rougher surface. Figure 2 compares the detailed mean charge density calculated using different methods.

As can be seen from table 1, table 2 and figure 2, the second-order correction term significantly improves the accuracy and also gives a reasonable capacitance variance estimate. The good agreement between Monte Carlo and SIE II for various roughnesses suggests that the stochastic integral equation method with the second order correction term is a promising approach. It is worth noting that the difference between smooth surface capacitance and the mean capacitance is approximately 10%. Hence the capacitance will be under-estimated if the surface roughness is neglected.

4.2 A three-dimensional example

The simple 3D example is a plate of zero thickness over ground plane. The ground plane is assumed to be smooth and the plate has a random rough profile, as shown in figure 3. The mean and the variance of capacitance calculated using different methods are compared in table 3 and 4, respectively. Again, the second-order correction significantly improves the accuracy of mean capacitance and gives an accurate estimate for the capacitance variance. It is worth noting that the algorithm and the implementation for 3D structures are the same as those for 2D structures.

5. Conclusions

We have demonstrated a stochastic integral equation method for calculating the mean and the variance of capacitance of both 2D and 3D structures. This method has two advantages: 1) It avoids the time-consuming Monte Carlo simulations; 2) It avoids the discretization of rough surface, which needs a much more refined mesh than the smooth surface. We are in the process of extending this method to the impedance extraction of 3D interconnects and packages with rough surfaces.

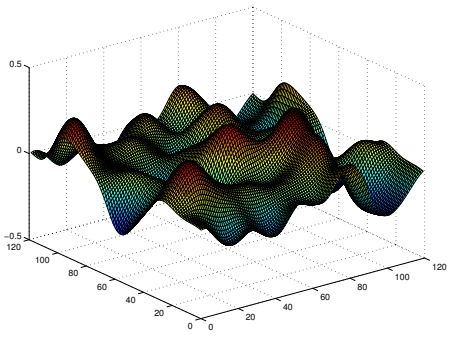

Figure 3: A zero-thickness plate with random profile. The correlation length is $0.2mm$ and standard deviation is $0.1mm$. The size of the nominal smooth plate is $1 \times 1mm$. The smooth ground plane is not included in this picture. The distance between nominal smooth plate and the ground plane is $0.5mm$.

Table 1: Mean value of 2D capacitance calculated with different methods. Unit:pF. η is the correlation length and σ is the standard deviation. Both are in *mm*.

η	σ	Smooth	Monte Carlo	SIE I	SIE II
0.2	0.1	57.68	61.42(5000run)	58.69	61.19
0.1	0.1	57.68	63.53(5000run)	59.80	64.72

Table 2: Variance of 2D capacitance by different methods. Unit:pF. η is the correlation length and σ is the standard deviation. Both are in *mm*.

η	σ	Monte Carlo	SIE II
0.2	0.1	3.72(5000run)	3.00
0.1	0.1	3.02(5000run)	2.24

Table 3: Mean value of 3D capacitance calculated with different methods. Unit:pF. η is the correlation length and σ is the standard deviation. Both are in *mm*.

η	σ	Smooth	Monte Carlo	SIE I	SIE II
0.2	0.1	56.599	62.656(4000run)	61.676	62.706
0.1	0.1	56.599	66.237(4000run)	63.850	65.471

Table 4: Variance of 3D capacitance calculated with different methods. Unit:pF. η is the correlation length and σ is the standard deviation. Both are in *mm*.

η	σ	Monte Carlo	SIE II
0.2	0.1	2.224(4000run)	2.011
0.1	0.1	1.194(4000run)	1.370

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