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AN FFT-BASED APPROACH TO INCLUDING NON-IDEAL GROUND PLANES IN A FAST 3-D INDUCTANCE EXTRACTION PROGRAM

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Abstract

Finite ground-plane conductivity can have an enormous effect on inductive coupling between signal lines. However, including non-ideal ground planes in 3-D inductance extraction programs is computationally expensive, as the ground plane must be finely discretized to insure the current distribution throughout the plane is accurately computed. This makes standard volume-element algorithms unsuitable because they require n^2 computation time and storage, where n is the number of filaments into which the ground plane is discretized. In this paper we show that by using a preconditioned iterative method combined with an FFT-based algorithm to compute the iterates, we can reduce the computation time to effectively $n \log n$, and substantially reduce required storage. Experimental results are presented to show that using the FFT-based approach is more than an order of magnitude faster than computing the iterates explicitly, even on problems with as few as a thousand volume-filaments.

1 Introduction

In [1], it was shown that an equation formulation based on mesh analysis can be combined with a GMRES-style iterative matrix solution technique to make a reasonably fast 3-D frequency dependent inductance and resistance extraction algorithm. Unfortunately, both the computation time and memory required for that approach grow faster than n^2, where n is the number of volume-filaments. This rapid growth in memory and computation makes the approach in [1] unsuitable when ground planes with finite conductivity are included because a large number of volume-filaments are needed to accurately model the ground plane current distribution. Although multipole algorithms have been applied in an attempt to reduce the memory and computation time required [2], there is a large overhead and a variety of approximations associated with the multipole algorithm. In this paper, we describe an FFT-based algorithm which avoids both the approximations and high overhead associated with the multipole algorithm, and is more than an order of magnitude faster than an explicit GMRES-style algorithm on problems with as few as a thousand volume-filaments.

2 The Mesh-Based Formulation

One approach to computing the frequency dependent inductance and resistance matrix associated with the terminal behavior of a collection of conductors involves first approximating each conductor with a set of piecewise-straight conducting sections. The volume of each straight section is then discretized into a collection of parallel thin filaments through which current is assumed to flow uniformly. The interconnection of these current filaments can be represented with a planar graph, where the n nodes in the graph are associated with connection points between conductor segments, and the b branches in the graph represent the current filaments into which each conductor segment is discretized.

To derive a system of equations from which the resistance and inductance matrix can be deduced, we start by assuming the applied currents and voltages are sinusoidal, and that the system is in sinusoidal steady-state. Following the partial inductance approach in [3, 4], the branch current phasors can be related to branch voltage phasors (hereafter, phasors will be assumed and not restated) by

$$ZI_b = V_b, \quad (1)$$

where $V_b,\ I_b \in C^b$, b is the number of branches (number of current filaments), and $Z \in C^{b\times b}$ is the complex impedance matrix given by

$$Z = R + j\omega L, \quad (2)$$

where ω is excitation frequency. The entries of the diagonal matrix $R \in \Re^{b\times b}$ represent the dc resistance of each current filament, and $L \in \Re^{b\times b}$ is the dense matrix of partial inductances Specifically,

$$L_{i,j} = \frac{\mu_0}{4\pi} \int_{filament_i} \int_{filament_j} \frac{l_i(X_i) \cdot l_j(X_j)}{|X_i - X_j|} d^3x_i d^3x_j, \quad (3)$$

where $X_i, X_j \in \Re^3$ are the positions in filament i and j respectively, and $l_i, l_j \in \Re^3$ are the unit vectors in the direction of current flow in filaments i and j.

Now assume that sources attached to the conductor system's terminals generate explicit branches in the graph representing the discretized problem. Kirchoff's voltage law, which implies that the sum of branch voltages around each mesh (a mesh is any loop of branches in the graph which does not enclose any other branches) in the network is represented by

$$MV_b = V_s \quad (4)$$

where V_b is the vector of voltages across each branch, except for the source branches, $V_s \in \Re^m$ is the mostly zero vector of source branch voltages, and $M \in \Re^{m\times b}$ is the mesh matrix, where m is the number of meshes.

The mesh currents, that is the currents around each mesh loop, satisfy

$$M^t I_m = I_b, \quad (5)$$

where the superscript t denotes matrix transpose, and $I_m \in \Re^m$ is the vector of mesh currents. Note that one of the entries in the mesh current vector will be identically equal to the source branch current. Combining (5) with (4) and (1) yields

$$MZM^t I_m = V_s. \quad (6)$$

The complex admittance matrix which describes the terminal behavior of the conductor system, denoted Y_r, can by derived from (6) by noting that

$$\tilde{I}_s = Y_r \tilde{V}_s, \quad (7)$$

where $\tilde{I}_s$ and $\tilde{V}_s$ are the vectors of source currents and voltages. Therefore, to compute the i^{th} column of Y_r, solve (6) with a V_s whose only nonzero entry corresponds to $\tilde{I}_{s_i}$, and then extract the entries of I_m associated with the source branches.

The standard approach to solving the complex linear system (6) is Gaussian elimination, but the cost is $O(m^3)$ operations. For this reason, iterative methods like GMRES [5] are often used. Such methods are still computationally expensive when applied to solving (6), because each iteration requires forming the product $MZM^t I_m$ which, as MZM^t is dense, requires $O(m^2)$ operations.

3 FFT Acceleration

It is possible to reduce the computational cost of calculating $MZM^t I_m$, and reduce the memory required to store MZM^t, by exploiting the fact that ground planes are typically discretized uniformly. Consider writing the matrix MZM^t as

$$MZM^t = \begin{bmatrix} T & F \\ F^t & D \end{bmatrix} \quad (8)$$

where $T \in C^{k\times k}$ represents the effects of ground plane mesh currents on ground plane mesh voltages, $F \in C^{k\times l}$ represents the effect of meshes outside the ground plane on mesh voltages in the ground plane, and $D \in C^{l\times l}$ represents the effect of mesh currents outside the groundplane on mesh voltages outside the groundplane. If the ground plane is discretized uniformly, then the elements of T are functions only of relative mesh positions; i.e. T is block Toeplitz with Toeplitz blocks. Thus the part of $MZM^t I_m$ associated with the ground plane mesh interactions is a 2D linear discrete convolution, which can be performed in $O(k \log k)$ operations by use of the Fast Fourier Transform (FFT). Therefore, the cost of the entire matrix-vector product will be $O(k \log k) + O(l(k + l))$. Also, as T, being Toeplitz, can be represented implicitly, the total memory required is $O(l(k + l))$. Note that this approach is only effective if the structures outside the ground plane are simple, as then l is much smaller than k.

It is desirable to accelerate the convergence of the GMRES algorithm by using preconditioning. We apply GMRES to the problem

$$MZM^t Py = V_s \quad (9)$$

and then obtain $I_m = Py$. GMRES will converge quickly if $MZM^t P$ is close to the identity matrix. An effective preconditioner is

$$P = \begin{bmatrix} K^{-1} & 0 \\ 0 & D^{-1} \end{bmatrix} \quad (10)$$

where $K \in C^{k\times k}$ is a doubly circulant matrix obtained from T by copying the central diagonals, similar to the approach discussed in [6]. We assume that $D \in C^{l\times l}$ is small and therefore easily inverted. As K is circulant, its inverse can also be computed quickly by use of the FFT.

Finally, we note that the matrix-vector product associated with the interactions in an irregular planar collection of conducting segments can be accelerated

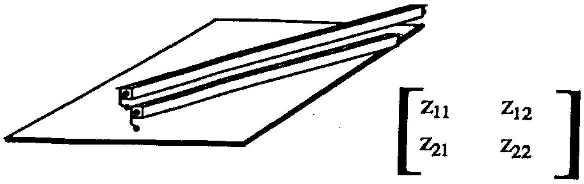

Figure 1: Example problem of two bars over a ground plane.

by the FFT technique. It is simply necessary to embed the irregular geometry into a larger regular rectangular region (thus introducing fictitious zero mesh currents in the part of the rectangular region with no conductors) before performing the FFT. The inefficiency introduced by including fictitious mesh currents is likely to be insignificant compared to the savings in using the FFT if the conductors cover a significant fraction of the rectangular region.

4 Results

To demonstrate both the importance of finite ground plane conductivity and the effectiveness of the FFT-accelerated iterative algorithm, consider the example in Figure (1). In this example, two conductors are placed parallel to each other, just above a ground plane with conductivity 5.8×10^7 mho/m. We extract the equivalent circuit impedance matrix elements z_{11}, z_{12} and z_{22} by applying unit voltages to each of the conductors in succession. The return path for current in each of the conductors is through the ground plane. To accurately model the current distribution, the ground plane was discretized into a 32×32 array of filaments. At low frequencies, the real part of the current is dominant. Figure (2) shows the real part of the computed ground plane current distribution at 100 Hz. At higher frequencies, say 10 MHz, the imaginary part of the current dominates, and is shown in Figure (3). Notice that at low frequencies the current spreads through the ground plane, but at high frequencies the current is concentrated under a conductor. The effect on the mutual inductance between conductors is dramatic, as shown in Figure (4). At high frequencies the inductance is more than an order of magnitude smaller than its value at low frequencies

To demonstrate the slow computation growth of the FFT-accelerated algorithm, the ground plane in Figure (1) was progressively refined, to make problems with successively more mesh current unknowns. As the plot in Figure (5) clearly indicates, for a ground-

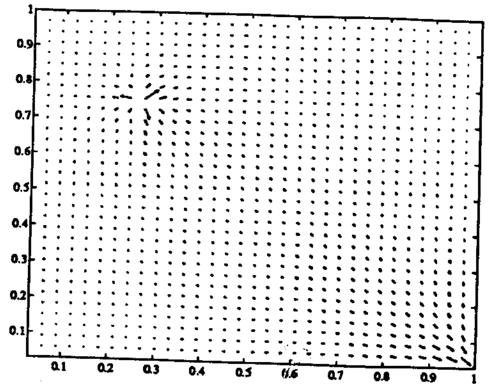

Figure 2: Real part of the ground plane current at 10^2 Hz.

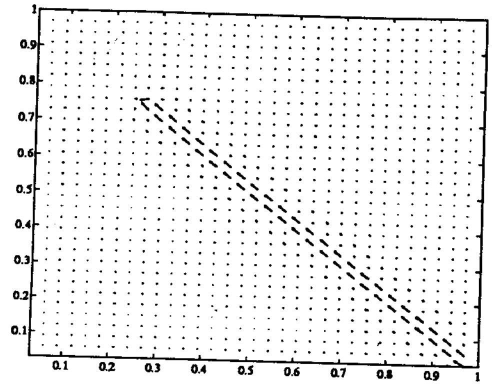

Figure 3: Imaginary part of the ground plane current at 10^7 Hz.

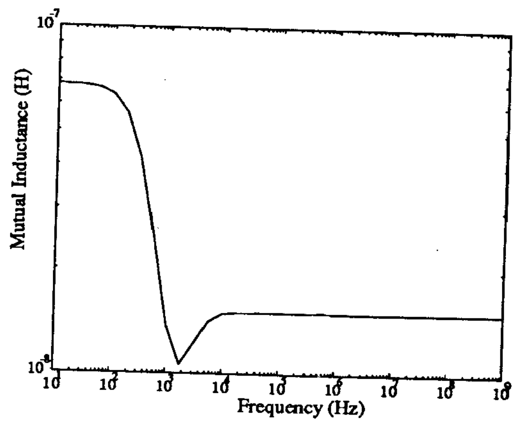

Figure 4: Mutual inductance between the bars of Figure (1).

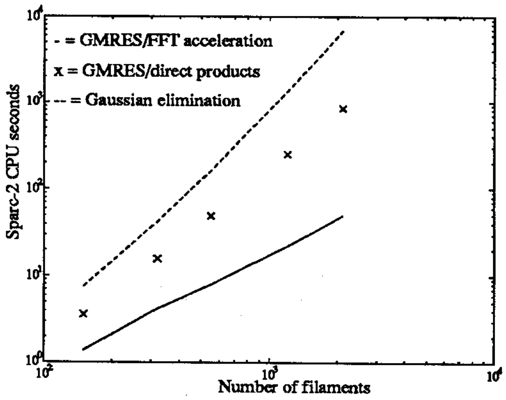

Figure 5: CPU time per frequency point for the example of Figure (1).

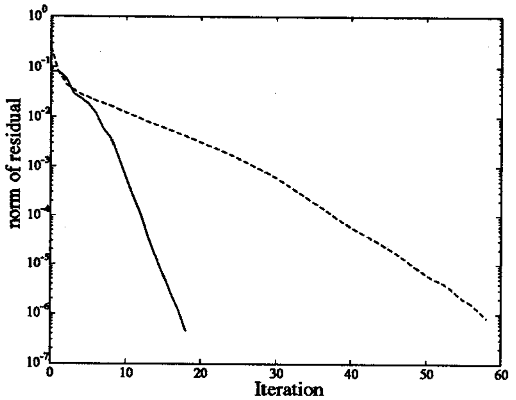

Figure 6: Norm of residual when GMRES is applied to the example of Figure (1). Dashed line shows norm of residual without preconditioning, solid line shows norm of residual when preconditioning is used.

plane discretized into a 32×32 array of filaments, the FFT-accelerated method is nearly twenty times faster than an iterative method with the matrix-vector products computed explicitly. Finally, Figure (6) shows that the preconditioner given above is effective in accelerating GMRES convergence.

5 Conclusions and Acknowledgments

In this paper we demonstrated that the time to compute the effects of ground plane resistance on coupling inductance can be substantially reduced through the use of an FFT-accelerated preconditioned GMRES algorithm. Future work is on extending the approach to more general ground plane structures and discretizations through the use of embedding and mapping techniques.

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