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Numerical study of self-field effects on dynamics of Josephson-junction arrays*

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We consider the influence of self-induced magnetic fields on dynamic properties of arrays of resistively and capacitively shunted Josephson junctions. Self-field effects are modeled by including mutual inductance interactions between every cell in the array. We find that it is important to include all mutual inductance interactions in order to understand the dynamic properties of the array, in particular subharmonic structure arising under AC current bias.

I. Introduction

To date, studies of Josephson junction arrays have generally neglected the effects of magnetic fields induced by currents flowing in the array. When these self-field effects are considered, it has often been assumed that the induced magnetic field in an array cell is produced by currents flowing only in that cell or its near neighbors [1,2]. Recently, we have shown that to correctly describe the static properties of vortices in arrays with no applied currents, it is necessary to include the inductive interactions of each array cell with *all* the currents flowing in the array [3].

It has been suggested that self-induced magnetic field effects may be responsible for subharmonic Shapiro step structure observed in rf-driven arrays [4]. Studies by Domínguez and José using an approximate treatment of the self-induced fields support this view [2]. In this paper, we show that subharmonic zero-field steps are observed in Josephson junction arrays when a complete treatment of mutual inductance interactions is considered, and that the I-V characteristics of the arrays are qualitatively different than when the inductive interactions are approximated. We have considered the usual case of strongly overdamped junctions (McCumber parameter $\beta_c = 0$) corresponding to SNS arrays, as well as critically damped systems ($\beta_c = 1$) which might be a better model for some arrays of SIS niobium junctions with resistive shunts.

II. Analysis

The general aspects of a mesh-based analysis of the Josephson array have been previously described in Ref. [3]. Briefly, we define two sets of variables: loop (mesh) currents, I, flowing in each array cell, and gauge-invariant phase differences ϕ across each junction. The difference in mesh currents between two adjoining cells must equal the current through the junction dividing the cells, which in the resistively-capacitively shunted junction (RCSJ) model is given by $I_J = \beta_c\dot{v}+v+\sin\phi$. Here, current is normalized to the junction critical current I_c, v is the voltage normalized to I_cR_n, time is normalized to the Josephson characteristic time $\tau_c = 2eR_nI_c/h$, and the damping is determined by $\beta_c = 2\pi R_nC/\tau_c$. In these units the time-evolution of the junction phase is simply $\dot{\phi} = v$. To relate the junction phases to the magnetic flux, we take a loop-sum of the phase differences around a cell so that $\sum\phi + 2\pi\Phi/\Phi_0 = 0$. The flux through a cell i, Φ_i, consists of an externally applied flux $\Phi^{\rm ext}$ and an induced part $\Phi_i^{\rm ind} = \sum_j L_{ij}I_j$, where the summation runs over all cells in the array and the matrix L is the inductance matrix. The diagonal entries of L are the self-inductances of the cells, and the off-diagonal

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entries the mutual inductances. The models of Ref. [1,2] correspond to setting most of the L_{ij} to zero. In this paper we retain all terms in L.

Applied currents are injected uniformly into each node along one side of the array. These injected currents can be modeled by adding additional meshes corresponding to current sources. Since the current through these meshes is known, the variables corresponding to the current source meshes can be eliminated from the system of equations that describes the augmented array circuit. This analysis produces a nonlinear system of differential-algebraic equations which can be discretized in time by standard techniques [5]. The linear systems which result from time-discretization of the differential equations can be solved either by LU-decomposition (for small, non-stiff systems) or through use of the FFT-accelerated iterative algorithm presented in Ref. [3].

III. Results

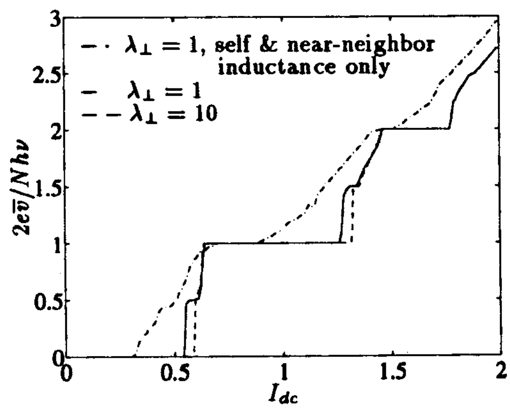

Figure 1: *Zero-field I-V characteristics of AC-driven* 11×11 *array,* $\beta_c = 0.0$

Figs. 1 and 2 show results of simulations of $N \times N$ junction arrays driven with an AC current of frequency $0.1/\tau_c$ and amplitude I_c per node, in zero applied magnetic field. The DC current injected into each node, I_{dc}, was varied between 0 and $2I_c$. Results for the overdamped array ($\beta_c = 0$, $N = 11$) are shown in Fig. 1. The solid curve corresponds to the full inductance matrix, and an effective penetration depth $\lambda_\perp = 1$ ($\lambda_\perp = 2\pi\mu_0 I_c/\Phi_0$ in units of the lattice spacing p). The first and second giant (integer) Shapiro steps at $V_1 = Nh\nu/2e$ are clearly visible, as are subharmonic steps at $\frac{1}{2}V_1$ and $\frac{3}{2}V_1$. When $\lambda_\perp$ is increased to 10, the order of the system size, the subharmonic structure essentially disappears, as can be seen from the dashed curve of Fig. 1. When the inductance matrix is approximated by retaining only self and nearest-neighbor inductances, a qualitatively different I-V characteristic is obtained, as shown in the dashed-dotted curve of Fig. 1.

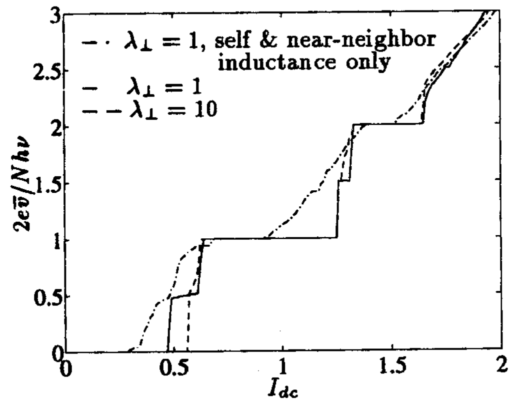

Figure 2: *Zero-field I-V characteristics of AC-driven* 11×11 *array,* $\beta_c = 1.0$

Fig. 2 shows results for a critically damped ($\beta_c = 1$) array. The effect of the induced fields is essentially the same as in the $\beta_c = 0$ case. The main distinction is that the addition of the capacitive term to the array dynamics broadens the subharmonic $\frac{1}{2}$ step.

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