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Convergence properties of Relaxation versus the Surface-Newton Generalized-Conjugate Residual Algorithm for Self-consistent Electromechanical Analysis of 3-D Micro-Electro-Mechanical Structures

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Abstract

In this paper, a relaxation and a surface-Newton method for self-consistent 3-D electromechanical analysis are theoretically and experimentally compared.

1 Introduction

Electrostatic sensors and microactuators are typically controlled by applied voltages which create electrostatic forces that deform the structure. Therefore, accurately analyzing the performance of these sensors and actuators requires self-consistent electromechanical analysis. However, self-consistent electromechanical analysis is a difficult computational problem because the discretization grid used must track the electrostatically deformed boundaries of the structure.

Self-consistent electromechanical analysis of complicated three-dimensional structures can be performed by combining a fast multipole-accelerated scheme for electrostatic analysis with a standard finite-element method for mechanical system analysis. There are two approaches for combining these analyses, one using a straight-forward relaxation scheme, and a second based on a surface-Newton method combined with a matrix-free generalized conjugate residual based solver (SNGCR). In this paper, the convergence properties of these two methods are examined. In particular, we show that relaxation will converge if the applied voltage is small enough, or if Young's modulus is large enough, but will diverge otherwise. We also show by example that although the SNGCR algorithm is guaranteed to converge only given a sufficiently close initial guess, it converges much more frequently than relaxation.

2 Self-Consistent Electromechanical Analysis Algorithms

Mechanical analysis programs solve discretized force equilibrium equations to determine final structure displacements given exterior deformation forces or pressures. For electromechanical analysis, the system of equations which must solved can be written in implicit

form as

$$d = F_M(x_o, F_E(d + x_o, V)) \tag{1}$$

where d is the unknown vector of discretized structure displacements, x_o is the vector of representing the initial structure position. Here, F_E denotes solving Laplace's equation to derive the electrostatic force given the structure position and applied potential V, and F_M denotes solving force-equilibrium equations to yield the structure displacements.

The simplest scheme for solving (1) is to first use a standard electrostatic analysis program to compute the forces on the structure, and then use the computed forces as input to a standard mechanical analysis program to compute the deformed structure. Then, the electrostatic forces can be recomputed on the deformed structure, and these new forces used to redeform the structure, and the process repeated until the forces and deformation converges. Such a relaxation process is *not* guaranteed to converge, and to examine its convergence properties we consider the relaxation algorithm's mathematical formulation

$$d^{k+1} = F_M(x_o, F_E(d^k + x_o, V)), \tag{2}$$

where k is the relaxation iteration index.

The relaxation will converge for a sufficiently close initial guess if

$$\left| \frac{\partial F_M}{\partial F_E} \frac{\partial F_E}{\partial d} \right| < 1. \tag{3}$$

Physically, $\frac{\partial F_E}{\partial d}$ is the change in electrostatic force due to structure displacement, and is therefore proportional to V^2. Also, $\frac{\partial F_M}{\partial F_E}$ is the amount which a structure deforms due to a change in applied force, and is inversely proportional to the Young's modulus (or stiffness) of the structure material. Therefore, (3) will be satisfied if either the material is stiff enough or if the applied voltage is small enough.

It is also possible to solve (1) using a Newton-like method, but rather than applying Newton's method to solving (1) directly, consider that once the displacement of the structure surface is known, both the surface electrostatic force and the structure's interior displacements can be determined by decoupled electrostatic and mechanical analysis. This suggests that the dimensionality of the coupled problem can be reduced from 3-D to 2-D, where only surface variables are involved in the coupled equations. Therefore, we can write a surface-Newton iteration equation as

$$F_s(d_s{}^k) + \frac{\partial F_s}{\partial d_s}(d_s{}^{k+1} - d_s{}^k) = 0 \tag{4}$$

where d_s is the vector of surface displacements, k is the iteration index, and

$$F_s(d_s{}^k) = d_s{}^k - Surf\left[F_M(x_{os}, F_E(d_s{}^k + x_{os}, V)\right]. \tag{5}$$

Here, x_{os} is used to denote the surface of the initial structure and the function $Surf$ extracts d_s from d.

A matrix-free GCR iterative method can then be used to solve (5), in which case each GCR iteration involves forming a matrix-vector product. The matrix-vector product can be computed using finite-differences, hence avoiding explicitly forming $\frac{\partial F_s}{\partial d_s}$.

3 Numerical examples

To show that the above analysis lends some practical insight, consider the two bar example in figure 1, which is intended to model a deformable mirror. In the experiments below, both the relaxation algorithm and the surface-Newton GCR algorithms were implemented using the FASTCAP [2] electrostatic analysis program and the finite-element mechanical analysis program ABAQUS [1]. In figure 2, we show that the relaxation algorithm converges more slowly and eventually diverges when the potential difference between the bars is increased. In figure 3, we show that making the material more flexible than silicon (Aluminum Oxide has a lower Young's modulus) also leads to nonconvergence. In figures 4 and 5, we show that applying Newton's method to a surface formulation of (1), and then solving the so-generated linear system by a matrix-free generalized conjugate direction method, leads to a more robustly converging algorithm. The algorithm, denoted SNGCR, is also faster than the relaxation method when both converge, as shown in figure 6.

4 Conclusions and Acknowledgments

In this paper it is shown that efficient electromechanical analysis can be performed by combining a standard finite-element based mechanical analysis program with a fast boundary-element based electrostatic solver. Also, we demonstrated that our surface/Newton-GCR algorithm is faster and more robust than the simpler relaxation scheme.

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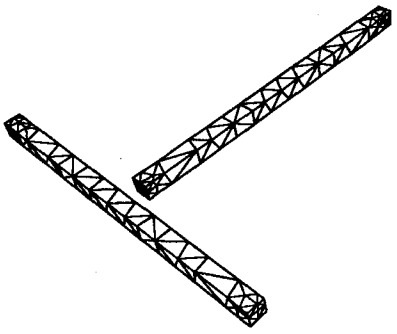

Figure 1: Silicon bars at different potentials

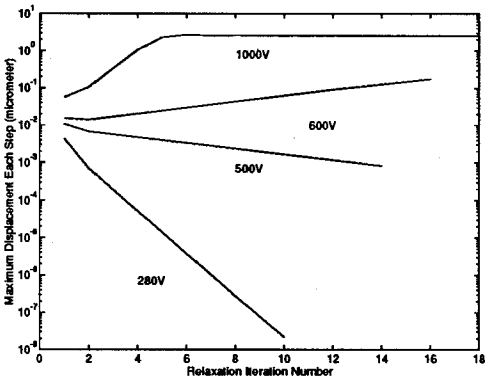

Figure 2: Relaxation convergence characteristics at different voltages

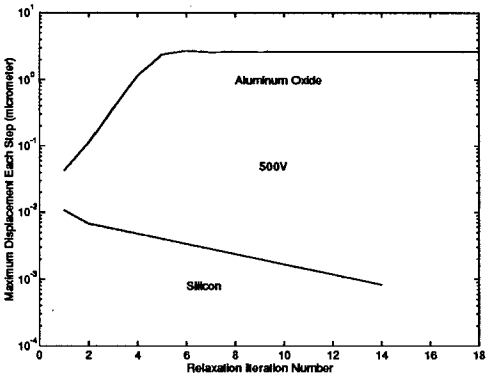

Figure 3: Relaxation convergence characteristics for different materials

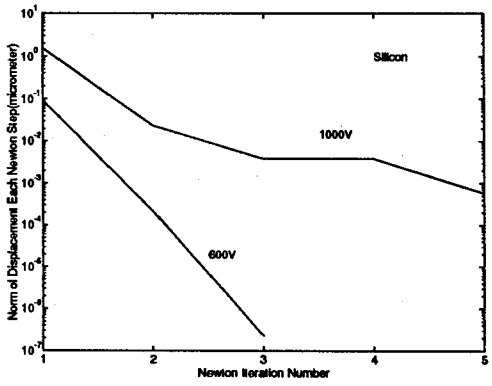

Figure 4: SNGCR convergence characteristics at different voltages

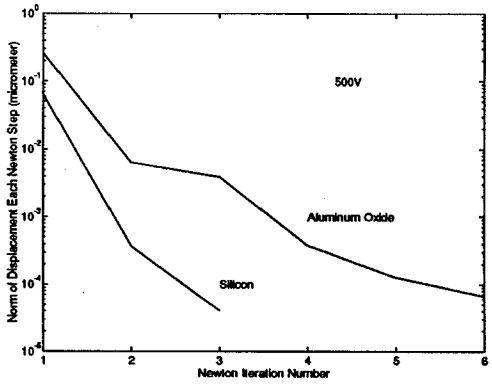

Figure 5: SNGCR convergence characteristics for different materials

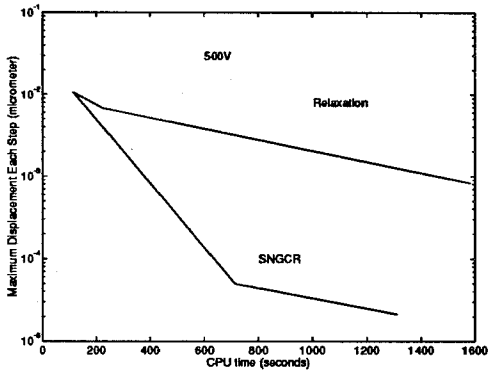

Figure 6: Relaxation and SNGCR convergence characteristics