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Extraction Techniques for Signal Integrity Analysis of 3-D Interconnect *

M. Chou, M. Kamon, K. Nabors, J. Phillips and J. White
Massachusetts Institute of Technology

Abstract

In this survey paper we describe recently developed algorithms for electromagnetic analysis of 3-D interconnect. The techniques described are fast enough to use on complicated 3-D interconnect structures and are sufficiently accurate that the results can be used to perform signal integrity analysis.

1 Introduction

The electrical performance of integrated-circuit packaging and on-chip interconnect is becoming progessively harder to predict. This is because new multilevel metal fabrication processes and advanced packaging techniques are generating three-dimensional interconnect structures of such geometric complexity that they are not easily be analyzed analytically. Predicting signal integrity problems, like too much cross-talk, is a particularly difficult given complicated 3-D interconnect. This is because problems like cross-talk can be the result of a large number of small coupling capacitances or small mutual inductances, each of which must be accurately computed. And in addition, cross-talk may depend critically on the interaction between the interconnect and attached driving circuitry.

Since the introduction of PEEC methods [9], the dominate approach to accurate coupled interconnect-circuitry analysis was modeling the interconnect with a densely coupled equivalent circuit derived using the method-of-moments [4]. PEEC-generated equivalent circuits were then combined with the nonlinear drivers and receivers and used as input to a circuit simulation program. And since circuit simulators use Gaussian elimination, the computational cost of that entire approach grews as n^3, where n was the number of circuit elements in the interconnect model. Therefore, PEEC-plus-circuit-simulation methods were simply too computationally expense to use for modeling complicated 3-D interconnect.

The above n^3 approach for coupled interconnect-circuitry analysis have recently been replaced with a variety of near linear-time, or nearly order n, algorithms which combine PEEC methods with model-order reduction and fast solvers. Such algorithms are a result of three key developments: the introduction of reduced-order modeling for interconnect, as in AWE [1, 2]; the development of Lanczos-based methods for constructing reduced-order models using only matrix-vector products [3]; and the development of fast-multipole and precorrected-FFT algorithms for rapidly computing dense matrix-vector products associated with discretized integral equations [6, 5]. In this paper we survey recent work in the topic with which we are most familiar, the fast-multipole and precorrected-FFT methods. We start in the next section by describing integral formulation for purely electrostatic (just capacitance), electroquasistatic (distributed resistance and capacitance), and magnetoquasistatic (distributed resistance and inductance) analysis. In section three we briefly describe the main ideas behind fast-multipole and precorrected-FFT methods. Results are given in section four and conclusions in section five.

2 Integral Formulations

Discretizations of the integral formulations of electrostatic, electroquasistatic, and magnetoquasistatic analyses all lead to dense matrix problems, where the entries in the dense matrix are associated with $1/r$ kernals. This commonality makes it possible to use the same approach to reduce the matrix-vector product time in all three problems.

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2.1 Electrostatic and Electroquasistatic Analysis

For conductors in a uniform dielectric medium, the self- and coupling-capacitances can be determined by solving the integral equation

$$\psi(x) = \int_S \sigma(x') \frac{1}{4\pi\epsilon_0 \|x - x'\|} dS', \qquad (1)$$

where S union of conductor surfaces, $\sigma(x')$ is the unknown conductor surface charge, and $\psi(x)$ is the given conductor potential. To numerically compute σ the conductor surface is discretized into n small panels. It is then assumed that on each panel i, a charge, q_i, is uniformly distributed. Finally, a system of equations is generated by insisting that (1) be satisfied at the center of each conductor panel. This leads to a system of equations of the form

$$\boldsymbol{P}Q = \Psi \qquad (2)$$

where $Q, \Psi \in \Re^n$ are the vector of panel charges and panel center potentials, and $\boldsymbol{P} \in \Re^{n \times n}$ is the matrix of potential coefficients given by

$$P_{i,j} = \int_{panel\ j} \frac{1}{4\pi\epsilon_0 \|x_i - x'\|} dS'. \qquad (3)$$

Following a similar derivation [10], electroquasistatic analysis yields a discretized integral equation of the form

$$-4\pi\tau \frac{\partial}{\partial t}\Psi = \boldsymbol{P}\Psi_n + PJ^{ext}, \qquad (4)$$

where $\boldsymbol{P}$ is as given in (3), and $\Psi_n \in \Re^N$ represents $(\frac{\partial \psi}{\partial n})$ at the n panels. Equation (4) simply states that currents from within the conductor, ψ_n, and from external sources, J^{ext}, both serve to charge or discharge the conductor surfaces, which then causes the potential everywhere to change in time according to the superposition integral.

2.2 Magnetoquasistatic Analysis

In the case of magnetoquasistatic analysis, used for extracting inductances, the conductor current density, J, satisfies $\nabla \cdot J = 0$ and for any point x in the conductor,

$$\frac{1}{\sigma} J(x) + \frac{j\omega\mu}{4\pi} \int_{V'} \frac{J(x')}{\|x - x'\|} dV' = -\nabla\psi(x), \qquad (5)$$

where here ψ is a scalar potential, σ is the conductivity, μ is the magnetic permeability, ω is the frequency of interest, and V' is the conductor volume.

To numerically compute J, the conductor volume is discretized into b filaments, and in each filament the conductor current is assumed constant. A system of equations for the filament currents, which are denoted by the vector I_b, is then generated by insisting that at filament intersection points, the directed sum of currents associated with the intersecting filaments is zero. In addition, the filament currents must satisfy

$$ZI_b = (R + j\omega L)I_b = V_b, \qquad (6)$$

where V_b, $I_b \in C^b$, b is the number of branches (number of current filaments), $Z \in C^{b \times b}$ is the complex impedance matrix, $R \in \Re^{b \times b}$ is the diagonal matrix whose elements are associated with the dc resistance of each current filament, and $L \in \Re^{b \times b}$ is the dense matrix of partial inductances [9]. Specifically,

$$L_{i,j} = \frac{\mu_0}{4\pi} \int_{filament_i} \int_{filament_j} \frac{l_i(X_i) \cdot l_j(X_j)}{|X_i - X_j|} d^3x_i d^3x_j, \qquad (7)$$

where $X_i, X_j \in \Re^3$ are the positions in filament i and j respectively, and $l_i, l_j \in \Re^3$ are the unit vectors in the direction of current flow in filaments i and j.

Using mesh analysis, it is possible to combine the current conservation constraint with (6) to yield

$$MZM^t I_m = V_s, \qquad (8)$$

where $I_m \in \Re^n$ is a vector of mesh currents, $M \in \Re^{n \times b}$ is the mesh matrix, V_s is the mostly zero vector of source voltages, and n is the number of meshes [8].

3 Accelerated Approaches

If an iterative algorithm is used to solve (2), (4) or (8), then each iteration of the algorithm will cost n^2 operations. This is because the matrices in (2), (4) and (8) are dense, and therefore evaluating candidate solution vectors involves a dense matrix-vector multiply. However, in all three cases, multiplying by the associated matrix is equivalent to evaluting a potential at n points due to n sources. This computation can be performed in order n operations using the fast multipole algorithm [6] or in $n \log n$ operations using a precorrected-FFT method [7].

As a brief explanation of how the fast multipole achieves its efficiency, consider the configuration, depicted in 2-D for simplicity, given in Figure 1. In the figure, the obvious approach to determining the electrostatic potential at the n_1 evaluation points from the n_2 point-charges involves $n_1 * n_2$ operations; at each of the n_1 evaluation points one simply sums the

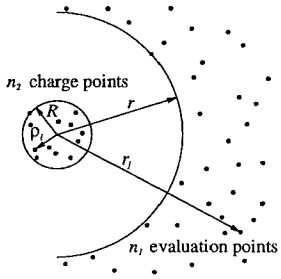

Figure 1: Exploiting charge or evaluation point clusters.

contribution to the potential from n_2 charges. An accurate approximation for the potentials for the case of Figure 1 can be computed in many fewer operations using *multipole expansions*, which exploit the fact that $r >> R$ (defined in Figure 1). That is, the details of the distribution of the charges in the inner circle of radius R in Figure 1 do not strongly effect the potentials at the evaluation points outside the outer circle of radius r. There is also a dual optimization. If the evaluation points are clustered inside the inner circle of Figure 1 and the charges are outside the circle of radius r, then it is possible to compute the evaluation point potentials in many fewer than $n_1 * n_2$ operations using *local expansions*. Local expansions exploit the fact that the potential due to distant charges varies slowly in space. Therefore, in this dual case, what can be ignored are the details of the evaluation point distribution.

Another approach to computing distant interactions is to exploit the fact that evaluation points distant from a cube can be accurately computed by representing the given cube's charge distribution using a small number of weighted point charges. If the point charges all lie on a uniform grid, then the Fast Fourier Transform (FFT) can be used to compute the potential at these grid points due to the grid charges. Specifically, Pq may be approximated in order $n \log n$ operations in four steps: (1) project the panel charges onto a uniform grid of point charges, (2) compute the grid potentials due to grid charges using an FFT, (3) interpolate the grid potentials onto the panels, and (4) directly compute nearby interactions. This process is summarized in Figure 2.

The precorrected-FFT method uses less memory than fast multipole algorithms, and is occasionally faster, as will be demonstrated below. But the major advantage of the precorrected-FFT methods is that the FFT is independent of the kernal, and so the approach can be combineds with modified Green's function methods for problems with ground planes or layered dielectrics [7].

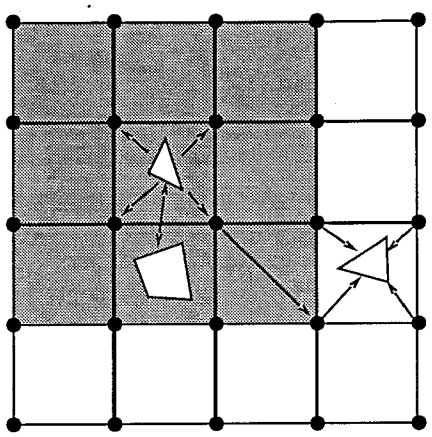

Figure 2: Representation of the four steps of the precorrected-FFT algorithm.

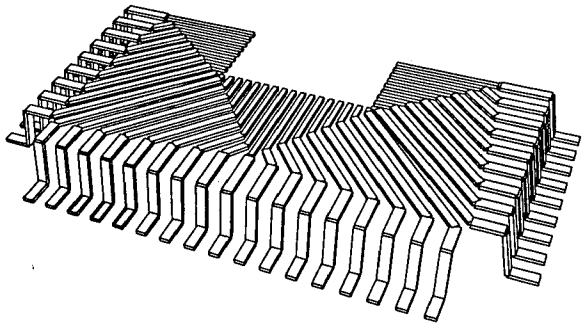

Figure 3: Half of a pin-connect structure. Thirty-five pins shown.

4 Results

The 35-pin package shown in Figure 3 can have sufficient inductive coupling to cause signal integrity problems. To demonstrate the efficiency of multipole acceleration, the CPU time versus number of filaments for direct versus multipole methods is plotted in Figure 4 (from FASTHENRY [8]). Very little accuracy is sacrificed to achieve the speed of the multipole-accelerated algorithms. The mutual inductance between neighboring pins and distance pins computed by direct methods are 0.0301 and 0.000208 respectively, and the results computed with multipole-accelerated algorithms are 0.0301 and 0.000207 respectively. As this example demonstrates, multipole-accelerated algorithms reliably compute small coupling inductances, so these techniques are suitable for signal integrity analysis.

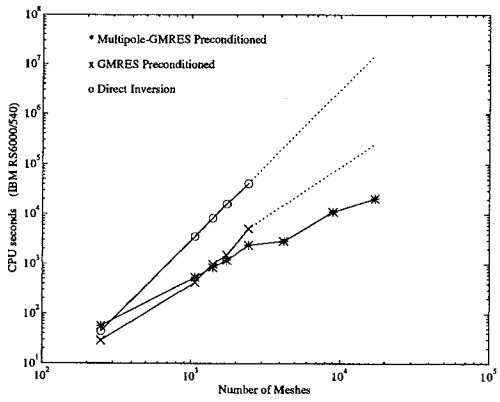

Figure 4: CPU times on an IBM RS6000/540.

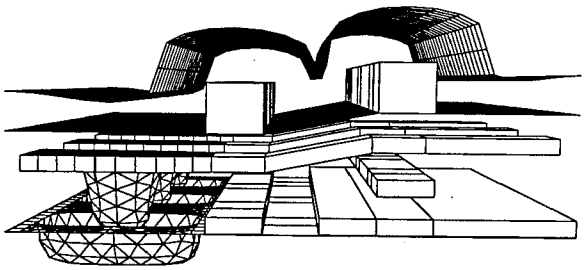

Figure 5: Neighboring DRAM cells.

As an example of capacitance extraction, consider the model of neighboring dynamic memory cells shown in Figure 5. The multipole-accelerated program FASTCAP [5] computes the capacitance matrix to one percent accuracy in 17 minutes on an IBM RS6000/540, but the direct factorization approach would take 1300 minutes.

The following table gives a comparison of the precorrected-FFT algorithm with the multipole-based code FASTCAP[5] on several realistic problems. Numbers in the table are the ratio of precorrected-FFT to FASTCAP CPU time or memory.

Example	Time	Memory	Product
bus crossing	0.59	0.26	0.15
via	2.26	0.37	0.84
DRAM cell	0.88	0.73	0.64

Note that the precorrected-FFT method can be as much as 40% faster and can use as little as one fourth the memory of FASTCAP[5]. Of course, this small performance improvement is not the most aspect of precorrected-FFT methods; their key advantage over multipole-accelerated algorithms is its ability to use general kernals.

5 Conclusions and Acknowledgements

Combining discretized integral equations with multipole or precorrected-FFT accelerated iterative methods leads to very efficient and accurate programs for determining the electromagnetic interactions in 3-D structures. Such tools make it possible to perform much more complete signal integrity analysis than previously possible, as it is computationally feasible to include many more coupling conductors. The authors have would like to thank all the writers mentioned in the references, we have valued their comments and suggestions over the years.

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