Efficient Reduced-Order Modeling for the Transient Simulation of Three-dimensional Interconnect

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Abstract

Multipole-accelerated surface-volume methods have proved to be very efficient techniques for delay and cross-talk simulation of three-dimensional integrated circuit interconnect. However, to be efficiently combined which transistor circuitry in a SPICE-level simulation, reduced-order models which have accurate low-frequency behavior must be constructed. Asymptotic Waveform Evaluation (AWE) or Pade-via-Lanczos (PVL) algorithms can not be used directly to construct the reduced-order models from the surface-volume formulation, because the formulation generates dense matrices which are too expensive to factor. In this paper we describe a two-level approach to efficiently generating reduced-order models with accurate low frequency behavior. First, reduced-order models which match Taylor series terms at $s = \infty$ are efficiently generated from the surface-volume formulation using an Arnoldi method, and then these fairly high-order models are used to efficiently construct lower-order models which match Taylor series terms at $s = 0$. Examples are given to demonstrate the accuracy of the resultant low-order model.

1 Introduction

When analyzing high-performance integrated circuit designs, it is well-known that the single lumped resistor-capacitor model of interconnect is insufficiently accurate. It has been suggested [1] that reasonably accurate electro-quasistatic, or transient interconnect, simulations could be performed by computing the time evolution of the electric field both inside and outside the conductors via a finite-difference discretization of Laplace's equation. More recently, a mixed surface-volume approach [2] was introduced, in which only the surfaces and the *interior* volume of the conductors are discretized, thereby avoiding the large, exterior domain mesh and computation. The capacitive coupling among the boundary elements is dense, but the computation of this dense interaction can be multipole-accelerated [3, 4, 5] when an iterative approach is used to solve the linear system to yield the solution at each timestep [2].

For such three-dimensional interconnect structures to be included along with the actual devices, both linear and non-linear, in a SPICE-type circuit simulator, it is necessary to construct low-order macromodels whose terminal behaviors essentially capture the complicated 3-D field interactions among the interconnect. The Asymptotic Waveform Evaluation [6] technique has been widely used to obtain reduced-order models from the lumped-element model of the interconnect, which result in sparse matrices that can be factored to match low-frequency moments of the transfer function. Very recently, a Pade-via-Lanczos algorithm [7] was shown to be more numerically robust process for computing the same reduced-order macromodel. This is achieved by exploiting the connection between the the Pade approximation and the Lanczos bi-orthogonalization process, which avoids the ill-conditioned problem of matching the moments explicitly.

For the reduced-order modeling of 3-D interconnect using the surface-volume formulation, it is impractical to factor the dense interaction matrix directly. Iterative solutions are more practical, especially when the matrix-vector product compuatation can be accelerated. In this paper, we show how to avoid the dense linear system solution all together by matching moments at infinite frequency, requiring only a small number of matrix-vector products. The resulting low-order model is represented by a small matrix, which can be factored directly to match several moments at zero frequency to further reduce the model order. The final reduced-order model is shown to be an highly accurate representation of the original system.

2 Background

In this section, we describe previous work on the time-domain surface-volume formulation of the transient interconnect problem and the Arnoldi-based model order reduction approach. The results recalled below will be used in the subsequent sections to help derive the nested model order reduction approach in the frequency-domain.

2.1 The Surface-Volume Formulation

For the transient interconnect problem, the system is assumed to be in the electro-quasistatic (EQS) regime. The scalar potential ψ satisfies

$$\nabla^2\psi(x) = 0, \qquad x \notin S, \tag{1}$$

which states that Laplace's equation holds everywhere except on conductor surfaces. Here S is the union of all conductor surfaces. Therefore, the potential ψ is related to the conductor surface charge density, ρ_s, through the superposition integral,

$$\frac{\partial\psi(x)}{\partial t} = \int_S \frac{1}{4\pi\epsilon\|x-x'\|}\frac{\partial\rho_s(x')}{\partial t}da'. \tag{2}$$

Charge conservation [8] at the surface yields the continuity condition $\frac{\partial\rho_s(x)}{\partial t} = J_{internal}(x) - J_{external}(x)$, where $J_{internal}$ and $J_{external}$ are the normal current densities taken just inside and just outside the conductor surface. The internal current obeys the constitutive relation $J_{internal}(x) = -\sigma\frac{\partial\psi}{\partial n}(x)$, where n is the outward normal to the surface S.

Combining (2) with charge conservation and the current constitutive relation, and noting that $J_{external} = 0$ for non-contact surfaces, we have

$$\begin{aligned} -4\pi\tau\frac{\partial\psi(x)}{\partial t} &= \int_S \tfrac{1}{\|x-x'\|}\tfrac{\partial\psi}{\partial n'}(x')da' \\ &\quad + \tfrac{1}{\sigma}\int_{S_{contact}} \tfrac{J_{external}(x')}{\|x-x'\|}da'. \end{aligned} \tag{3}$$

We now break up the conductor surfaces into N small tiles or panels. It is then assumed that on each panel l, the potential ψ_l, its normal derivative $\frac{\partial\psi}{\partial n}_l$, and the external current density J_l^{ext}, are all constants. A collocation scheme [9], in which (3) is enforced at the centroid in each of N panels, is used to generate a system of N equations. The result is a *dense* $N \times N$ linear system

$$-4\pi\tau\frac{\partial}{\partial t}\Psi = P\Psi_n + PJ^{ext}, \tag{4}$$

where $\tau = \sigma/\epsilon$ is the dielectric relaxation time. Ψ, Ψ_n, J^{ext} are N-vectors whose elements represent the potential, its normal derivative, and the external currents on the N panels, respectively. $P \in \Re^{N\times N}$ is a dense matrix whose elements are

$$P_{kl} = \frac{1}{a_l}\int_{panel_l}\frac{1}{\|x'-x_k\|}da', \tag{5}$$

where x_k is the centroid of the k^{th} panel, a_l is the area of the l_{th} panel.

For each conductor in a given problem, if the potential is known on the entire surface, Laplace's equation can be solved for the *interior* domain via finite-differences to yield Ψ_n everywhere *just* inside the surface. Let $\mathcal{X}$ be defined such that

$$\mathcal{X}\Psi = \Psi_n, \tag{6}$$

and applying $\mathcal{X}$ implies solving the interior problems. Using (6) in (4), and letting $D = P\mathcal{X}$,

$$-4\pi\tau\frac{\partial}{\partial t}\Psi = D\Psi + PJ^{ext}. \tag{7}$$

Since D is singular [2], the steady-state voltage Ψ is not uniquely specified by the external current J^{ext}. Instead, (7) will be reformulated as a differential-algebraic problem in Section 3, in which voltage sources are assumed instead of current sources.

2.2 Order Reduction using Arnoldi Iterations

Consider a linear, time-invariant system, such as a large linear circuit or 3-D interconnect, which is described by the system of first-order ordinary differential equations

$$\begin{aligned} \mathbf{x} &= \mathbf{A}\dot{\mathbf{x}} + \mathbf{b}u, \\ y &= \mathbf{c}^{\mathbf{T}}\mathbf{x}. \end{aligned} \tag{8}$$

Here, the vector $\mathbf{x}$ represent the circuit variables or the detailed internal voltages of the interconnect, the matrix $\mathbf{A}$ represents the detailed interations among internal elements, $\mathbf{b}u$ is the input excitation vector, and y is the output of interest. The state-space representation of (8) is

$$\begin{aligned} \mathbf{X} &= s\mathbf{A}\mathbf{X} + \mathbf{b}U, \\ Y &= \mathbf{c}^{\mathbf{T}}\mathbf{X}, \end{aligned} \tag{9}$$

where $\mathbf{X}, U$, and Y denote the Laplace transforms of $\mathbf{x}, u$, and y, respectively. The transfer function $H(s) = Y(s)/U(s)$ is then

$$H(s) = \mathbf{c}^{\mathbf{T}}(\mathbf{I} - s\mathbf{A})^{-1}\mathbf{b}. \tag{10}$$

In general, the size of the linear system can be of the order of tens of thousands, which is much larger than the number of input and output terminals, generally of the order 10. It is thus impractical to include the entire linear system (8) into the circuit simulator, such as SPICE. Instead, an N-port reduced-order model is needed which is simple enough to be included in a

circuit simulator and which is accurate enough from the perspective of terminal currents and voltages, *i.e.* matches the transfer function in (10).

It has been shown in [10] that an Arnoldi-based orthogonalization process can be used to construct an orthonormal basis for the Krylov subspace

$$\mathcal{K}_{\mathbf{k}}(\mathbf{A},\mathbf{b}) = \mathbf{span}\{\mathbf{b}, \mathbf{Ab}, \mathbf{A^2b}, \cdots, \mathbf{A^{k-1}b}\}. \quad (11)$$

After q steps, the Arnoldi algorithm returns a set of q orthonormal vectors, as the columns of the matrix $\mathbf{V}_q \in \Re^{m\times q}$, where m is the size of $\mathbf{A}$, and typically $q \ll m$. The reduced-order transfer function can then be constructed as

$$\begin{aligned} \tilde{H}(s) &= \tilde{\mathbf{c}}^{\mathbf{T}}(\tilde{\mathbf{I}} - s\tilde{\mathbf{A}})^{-1}\tilde{\mathbf{b}}, \\ \tilde{\mathbf{A}} &= \mathbf{V_q^T A V_q} = \mathbf{H_q}, \\ \tilde{\mathbf{b}} &= \mathbf{V_q^T b} = \|\mathbf{b}\|\mathbf{e_1}, \\ \tilde{\mathbf{c}}^{\mathbf{T}} &= \mathbf{c^T V_q}, \end{aligned} \quad (12)$$

where $\mathbf{H_q}$ is $q \times q$ upper Hessenberg. The transfer function of the reduced q^{th}-order system (12) can be obtained directly by using the eigen-decomposition $\mathbf{H_q} = \mathbf{S_q \Lambda_q S_q^{-1}}$, resulting in

$$\tilde{H}(s) = \sum_{k=1}^{q} \frac{\mu_k \nu_k}{1 - s\lambda_k} \quad (13)$$

The q^{th}-order transfer function has been shown in [10] to match $(q-2)$ derivatives, or moments, of the exact transfer function in (10) at $s = 0$, the low-frequency limit.

3 Algorithm

In this section, we describe an efficient algorithm for computing a reduced-order model for the transient interconnect problem which avoids solving large, dense linear systems.

Because the matrix D in (7) is singular, we will reformulate the problem into a differential-algebraic (DAE) system. This is done by using voltage sources instead of current sources, and then computing the resulting q-port frequency-dependent admittance matrix, which is then well-behaved near zero frequency. Assume that M of the N total surface panels described in Section 2.1 are connected to external voltage sources, whose potentials are thus known *a priori*. The N unkowns in (7) are now the $(N - M)$ floating potentials and the M externally supplied currents J_c^{ext}. The result is a system of differential equations with algebraic constraints. In frequency domain, the result is a system of equations

$$-P_{11}J_c^{ext} - D_{12}\Psi_f = (D_{11} - sI)\Psi_c, \quad (14)$$

$$-P_{21}J_c^{ext} + (sI - D_{22})\Psi_f = D_{21}\Psi_c, \quad (15)$$

where $D_{11}, D_{12}, D_{21}, D_{22}$ are partitions of the D matrix and P_{11}, P_{21} are partitions of the P matrix. The subscript 1 denotes panels in contact with voltage sources and the subscript 2 denotes the free-floating panels. Now let $b \in \Re^M$ be a vector of ones and zeros which selects a single input voltage source and effectively grounding the other sources. Then $\Psi_c = bu$, where u is the scalar input. Since the number of contact panels is typically much smaller than that of floating panels, P_{11} is a small matrix and can be inverted to perform a block LU factorization

$$-P_{11}J_c^{ext} - D_{12}\Psi_f = (d_1 + sd_s)u, \quad (16)$$

$$(sI - A)\Psi_f = (b_1 + sb_2)u, \quad (17)$$

where $d_1 = D_{11}b$, $d_2 = -b$, $b_1 = D_{21}b - P_{21}P_{11}^{-1}d_1$, $b_2 = -P_{21}P_{11}^{-1}d_2$, and $A = D_{22} - P_{12}P_{11}^{-1}D_{12}$.

It is now necessary to manipulate (17) into a form suitable for model-order reduction. We first expand Ψ_f as a sum of two power series in $(\frac{1}{s})$, and then recombine terms of like powers to obtain the result $(\frac{1}{s})$

$$\Psi_f = b_2 u + \left(\frac{1}{s}\right)\left(I - \left(\frac{1}{s}\right)A\right)^{-1} vu, \quad (18)$$

where $v = b_1 + Ab_2$. Let the transfer function be defined as

$$h(s) = \frac{c^T \cdot J_c(s)}{u(s)} \quad (19)$$

By combining (16), (18), and (19), we immediately get

$$h(s) = (k_0 + k_1 s) + \left(\frac{1}{s}\right) l^T \left(I - \left(\frac{1}{s}\right)A\right)^{-1} v, \quad (20)$$

in which the first term on the RHS can be computed exactly cheaply. The second term is expensive to compute since A is *large* and *dense* in our problem.

To match the moments of (20) at zero frequency directly using AWE [6] or PVL [7] would require solving the dense system several times, which is prohibitively expensive for large problems. We propose here an efficient, two-level approach for constructing a low-order model for the triplet $[l, A, v]$. In the first stage, the Arnoldi iteration is used to generate an

intermediate-order model $[l', A', v']$ by matching moments, or Taylor series terms, at $s = \infty$. Only a few matrix-vector products with A are required here, and this computation can be multipole-accelerated as in [2]. Enough moments are generated to ensure that the model also produces the exact solution at $s = 0$. For the transient-interconnect, or 3-D distributed RC problem, tenth-order models are adequate. In the second stage, the intermediate model is further reduced by moment-matching at $s = 0$. The matrix A', typically 10×10, is inverted explicitly, and this inverse is repeatedly applied to v' to produce moments about $s = 0$. Three moments generally give sufficient accuracy, and this final third-order model $[l'', A'', v'']$ can be very efficiently incorporated in a SPICE-like circuit simulator.

4 Preliminary Results

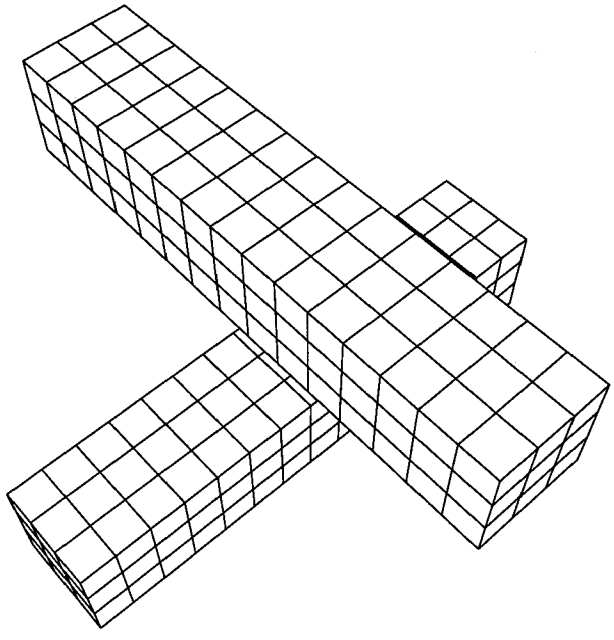

Figure 1: Four-port Bus Crossing Example

In this section we demonstrate the effectiveness of two-level model-order reduction technique by using the method to construct a reduced-order model for the four-terminal bus-crossing example shown in Figure 1. Each conductor is $5\mu m$ long and has a $1\mu m \times 1\mu m$ cross-section. The separation between them is $1\mu m$. It is assumed that the conductors are polysilicon, with $\rho = .01\Omega - cm$, and $\epsilon_r = 12$ everywhere, which imply $\tau = 2 \times 10^{-14}$. The conductor surfaces are discretized into a total of 396 square panels of equal area, 36 of which belong to the four end-faces, and each face is connected to an independent voltage source. The voltage source at one of the end-faces is a unit-ramp with a rise-time of 50τ, or 1 picosecond, and the other three end-faces are grounded. The resulting current flowing in each of the four terminals is computed from the frequency-domain transfer function, which can be computed exactly or from a reduced-order model. The reduced-order model obtained by matching 10 moments at infinite frequency is seen to produce the exact low-frequency limits of the exact transfer function. A further reduction of this 10^{th} order model to a 3^{rd} order model by matching 3 moments at zero-frequency is seen to produce virtually the identical time-domain response as the 10^{th}-order model. This is shown in Figures 2, 3. The exact transfer function was also computed by explicit eigen-decomposition of the 360×360 matrix A, and the absolute error produced by the 3^{rd}-order model in the time-domain shown in Figure 4.

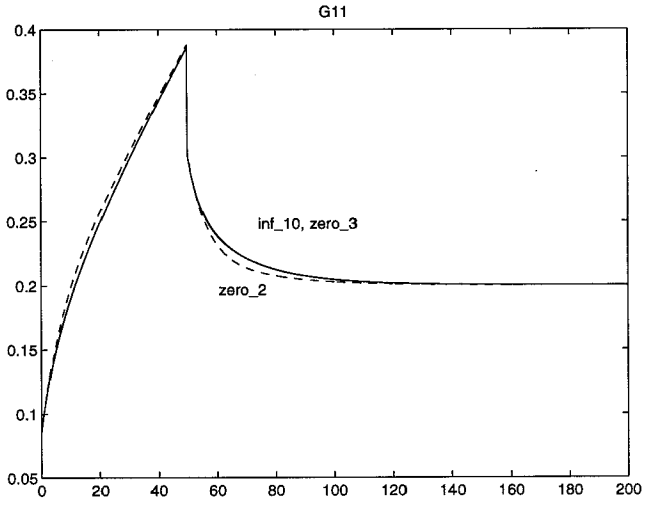

Figure 2: I_1 (unit=5 mA) vs. time (unit=2×10^{-14} s)

5 Conclusions

In this paper we described a two-level approach to efficiently generating accurate low-order models directly from three-dimensional interconnect. First, a multipole-accelerated Arnoldi method was used to generate tenth-order models by matching Taylor series terms in the transfer function at $s = \infty$. Then, the tenth-order models where used to generate three-order models which matched Taylor series terms in the transfer function at $s = 0$. Preliminary results from a

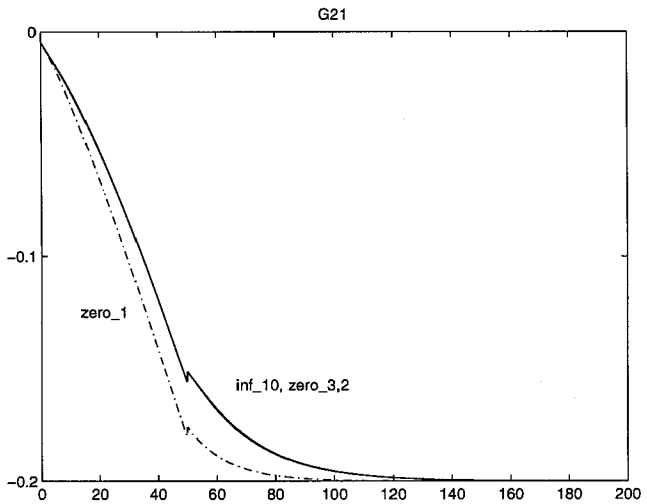

Figure 3: I_2 (unit=5 mA) vs. time (unit=2×10^{-14} s)

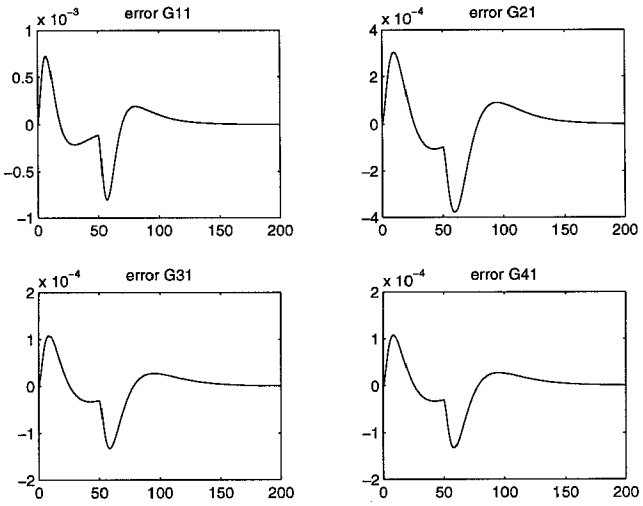

Figure 4: I_{err} (unit=5 mA) vs. time (unit=2×10^{-14} s)

cross-over example was examined to demonstrate the accuracy of the two-level approach. In particular, it was shown that the generated third-order model produced ramp response which were nearly indistinguishable from the exact ramp responses.

Future work is to examine a broader collection of examples, and to gain a better understanding of how to automatically pick the orders in each level of the algorithm.

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