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Two Optimizations to Accelerated Method-of-Moments Algorithms for Signal Integrity Analysis of Complicated 3-D Packages. *

Mattan Kamon†, Byron Krauter‡, Joel Phillips†, Lawrence T. Pileggi‡, Jacob White†

1 Introduction

The electrical performance of integrated-circuit packaging is becoming progessively harder to predict because advanced packaging techniques are generating complicated three-dimensional interconnect structures. Such structures are nearly impossible to analyze analytically, particularly when trying to evaluate signal integrity, and designers are becoming more reliant on accurate computer simulation tools. Signal integrity problems, like ground-plane noise, are particular hard to simulate because so much of the problem geometry must be included to achieve accurate results.

Multipole and precorrected-FFT accelerated Method-of-Moments techniques [1, 3, 2, 4, 8] are one of the few techniques that are fast enough to analyze signal integrity problems, so optimizing these techniques seem worthwhile even if the resulting optimizations are somewhat incremental. In this paper we describe two optimizations to accelerated method-of-moments simulation, the first is an even better preconditioner than presented in [4] and the second is several optimizations of the FFT-based convolution used in precorrected-FFT methods [8].

2 A New Preconditioner for Efficient Inductance Extraction

Iterative algorithms used to solve the dense systems of equations resulting from the integral equations of magnetoquasistatic analysis rely on preconditioning to insure fast convergence. The preconditioning matrix must be a good approximation to the inverse of the original system and be inexpensive to compute.

Various approaches to preconditioning for magnetoquasistatic analysis have been explored in [4, 5] which show that since the original system is positive definite, so must the preconditioner. The dominant technique presented involves deriving a positive definite sparsification of the partial inductance matrix.

Recently, a method has been proposed for stably approximating the partial inductance matrix to any degree of sparsification [6]. The central idea of this approach is to assume that the partial element conductor currents return at some finite and constant radius from their origin rather than from infinity. The coupling inductances within the "shells" of return current are shifted, while those outside become zero.

By choosing the radius small enough, the resulting matrix will be sparse enough that this shell approach can be used as a preconditioner. Conductor segments which are only partially outside the shell can be treated approximately. Table 1 compares using block-diagonal preconditioners to the shell preconditioner for various radii. Clearly, the shell preconditioner converges in many fewer iterations, however the overall execution speed up is not as dramatic since the preconditioner is more dense than the block-diagonal matrix.

It may be possible to improve the results for the small radii preconditioners by more accurately treating the conductor segments that are partially outside the current shells.

3 An Optimization for Precorrected-FFT Methods

When an iterative algorithm is used to solve the Method-of-Moments matrices associated with integral formulations of electrostatic and magnetoquasistatic analysis, the major cost of the algorithm is computing the dense matrix-vector products. A variety of sparsification techniques have been applied to rapidly compute the matrix-vector products, such as fast multipole algorithms [3] or precorrected-FFT methods [8].

The precorrected-FFT approach to computing distant interactions is to exploit the fact that evaluation points distant from a cube can be accurately computed by representing the given cube's charge distribution

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†Department of Electrical Engineering and Computer Science, Massachusetts Institute of Technology, Cambridge, MA, 02139. Fax: 617-258-7864. Kamon and Phillips: 617-253-1205, matt@rle-vlsi.mit.edu, jphill@rle-vlsi.mit.edu. White: 617-253-2543, white@rle-vlsi.mit.edu

‡Department of Electrical and Computer Engineering, University of Texas at Austin, Austin, TX, 78712, Fax: 512-471-5445, Phone: 512-471-0529, byron@cad.ece.utexas.edu, pileggi@cad.ece.utexas.edu

Preconditioner type		# iterations	non-zeros in precond	total CPU time
Block Diagonal	diagonal-of-L	1537	4613	1187
	cube-block	774	101671	729
Shell Current	$r = 0.25$	643	24829	590
	$r = 0.5$	474	184295	565
	$r = 0.75$	389	326579	595
	$r = 1.0$	349	432121	656

Table 1: Results of using various preconditioners to solve for the admittance of a coarse discretization of a 35 pin package at a high frequency. Cube-block refers to dividing space into cubes whose side lengths are roughly 1/8th of the package width. The radii are relative to the cube side length.

Example	Grid Order	Setup	Solve	CPU	Memory	Product
via	$p = 2$	0.58	0.22	0.28	0.28	0.078
	$p = 3$	0.57	0.61	0.61	0.37	0.23
woven5x5	$p = 2$	0.16	0.19	0.19	0.20	0.037
	$p = 3$	0.87	0.43	0.45	0.48	0.22
cube	$p = 2$	0.14	0.31	0.21	0.22	0.046
	$p = 3$	0.42	0.34	0.38	0.32	0.12
bus3x6	$p = 2$	0.29	0.10	0.12	0.16	0.019
	$p = 3$	0.30	0.24	0.25	0.2085	0.052
bus3x8	$p = 2$	0.19	0.16	0.16	0.17	0.027
	$p = 3$	0.50	0.25	0.27	0.27	0.074

Table 2: Comparison of FASTCAP and precorrected-FFT codes. Table entries are ratios of precorrected-FFT/FASTCAP times.

using a small number of weighted point charges. If the point charges all lie on a uniform grid, then the Fast Fourier Transform (FFT) can be used to compute the potential at these grid points due to the grid charges. Specifically, Pq may be approximated in order $n \log n$ operations in four steps: (1) project the panel charges onto a uniform grid of point charges, (2) compute the grid potentials due to grid charges, which is a three-dimensional convolution, using an FFT, (3) interpolate the grid potentials onto the panels, and (4) directly compute nearby interactions.

The precorrected-FFT method uses less memory and is much faster than even the optimized fast multipole algorithms used in programs like FASTCAP [2]. However, to achieve the best speed from precorrected-FFT methods, the three-dimensional convolution associated with computing grid charges from grid potentials must be optimized. For the results presented in [8], the three-dimensional convolution was computed using zero-padding combined with the general three-dimensional FFT routines from [7]. The convolution can be made nearly five times faster by combining three tricks specific to our problem.

The first trick for optimizing the use of the FFT to compute the 3-D convolution is to exploit the fact that the FFT's shuffle exchange and reexchange can be avoided when performing convolution. The second trick is to exploit the fact that computing direct convolution requires that the grid be zero-padded by a factor of two in each dimension before performing the transform. If the 3-D FFT is computed using a sequence of one-dimensional FFTs, then it is possible to avoid nearly half the one-dimensional transforms by ignoring lines of zeros. Finally, if the one-dimensional FFTs are reorganized within each dimension, then it is possible to insure better locality in data accesses which can improve workstation performance by nearly a factor of two on large problems.

In the following table, we compare the performance of the precorrected-FFT method to the fast multipole algorithm used in the FASTCAP program. We used second-order multipole expansions with FASTCAP and compared it with both the $2 \times 2 \times 2$ and $3 \times 3 \times 3$ grid in the precorrected-FFT method. The $2 \times 2 \times 2$ grid results were occasionally slightly less accurate than the FASTCAP results, but the $3 \times 3 \times 3$ grid results were almost always much more accurate.

With the optimized convolution, it was possible to compute all the coupling capacitances for one of the

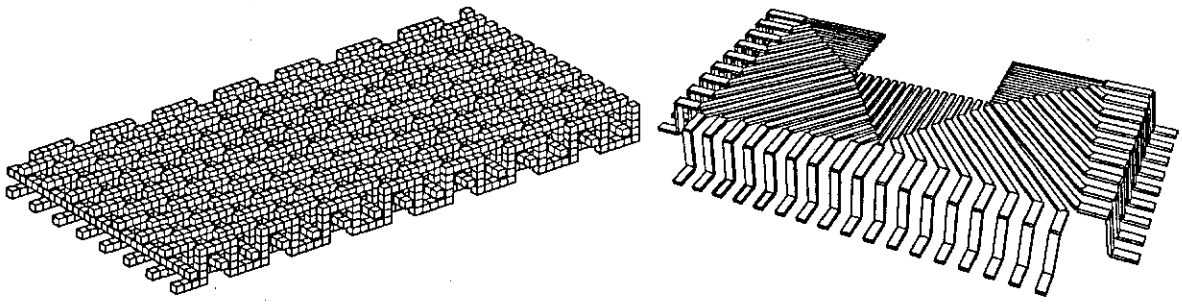

Figure 1: Three dimensional Woven Buss from Table 2 (actual discretization is 82,000 panels) and 35 pins of a cerquad package from Table 1.

interior conductors in the following woven buss problem. What is most impressive about the results is that the problem was discretized into nearly 100,000 panels but the matrix solution time was less than three minutes.

4 Conclusions

In this paper we presented two optimizations to accelerated Method-of-Moments algorithms. Although neither of the results provides astounding preformance improvement, we hope that the results are of interest because of the importance of having faster 3-D analysis techniques.

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