Proc. IEEE Workshop on Electrical Performance of Electronic Packaging, 10/96.

Mesh refinement strategies for capacitance extraction based on residual errors

Johannes Tausch and Jacob White
Research Laboratory of Electronics
MIT
Cambridge, MA 02139
FAX (617) 258-7864
tausch@mit.edu, white@rle-vlsi.mit.edu

Abstract— **We investigate mesh generation for capacitance calculations. The refinement scheme is controlled by an error estimator based on the residual of the solution calculated on a coarse grid. Numerical experiments are presented.**

I. Introduction

Engineering programs which compute electrostatic capacitances for complicated arrangements of conductors commonly set up the electrostatic potential u as a superposition of surface charges σ

$$u(x) = V\sigma(x) := \int_S G(x,y)\sigma(y)\, dS_y \qquad x \in \mathrm{R}^3\,, \tag{1}$$

where $G(x,y) = 1/4\pi|x-y|$ is the Green's function for the Laplacian in the three-space. For a specified potential f on the conductor surface(s) S, this approach leads to the integral equation of the first kind

$$V\sigma(x) = f(x)\,, \qquad x \in S\,. \tag{2}$$

The charge density is singular in regions where the conductor surface has edges and corners and is almost singular in regions of sharp curvatures. This calls for a finer discretization in these regions. The aim of this study is devise an adaptive algorithm that refines the grid based on the feedback of the solution calculated on a coarser grid.

Similar to most approaches taken in finite element calculations [1], [2], we base our estimator of the local error on the residual of the solution σ_h obtained on the coarse mesh. In the context of (1), the residual is defined by

$$r_h = V\sigma_h - f\,,$$

This can be motivated by the following relation between the residual and the discretization error $e_h = \sigma_h - \sigma$

$$r_h = V\sigma_h - V\sigma = Ve_h\,.$$

In the proper Sobolev space setting, the operator V in (2) has a continuous inverse [3]. This leads to an estimate of e_h in the mean-square norm in terms of the residual in the Sobolev-1 norm

$$\|e_h\|_0^2 \le c\,\|r_h\|_1^2 = c\,\left(\|r_h\|_0^2 + \|\nabla_S r_h\|_0^2\right)\,.$$

We further estimate the norms on the right hand side by applying the mid-point quadrature rule. The residual of the collocation solution vanishes at the midpoints and we are left with

$$\|e_h\|_0^2 \le c\sum_j \alpha_j s_j^2\,, \tag{3}$$

where α_j is the panel area, $s_j = |\nabla_S r_h(x_j)|$, ∇_S is the surface gradient, and c a constant independent of the grid.

The underlying idea of the refinement algorithm is to break up the panels that have the largest contribution to the error estimator (3). To control the number of panels in the refined grid, we specify the parameter $0 \le \gamma < 1$

Algorithm

1. solve (2) on a given grid
2. calculate the error indicators s_j
3. refine every panel k that satisfies

$$s_k^2\alpha_k \ge \gamma \max_j s_j^2\alpha_j$$

uniformly into four panels.

4. repeat steps 1. – 3.

II. Numerical Experiments

Here we present some preliminary results of the mesh refinement strategy discussed above. We have

This work is supported by the NSF and ARPA

tested the algorithm for the unit cube, an L-shaped domain and a structure consisting of two cubes. The tables compare the capacitances calculated with uniform refinement (setting $\gamma = 0$ in the algorithm above) with adaptive meshing ($\gamma = 0.5$).

III. Conclusions

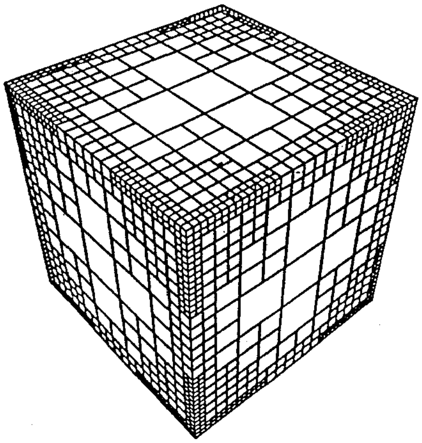
1. The surface gradient of the residual provides a mean to estimate the discretization error.
2. The grids produced by our algorithm allow accurate estimates of the capacitance with significantly fewer panels than uniform discretizations.
3. Adaptive procedures that minimize the discretization in other norms, like the energy norm, could result in even better grids and will be investigated in the future.

References

[1] Claes Johnson and Peter Hansbo, "Adaptive finite element methods in computational mechanics", *Comp. Meth. in Appl. Mech. Engrg.*, vol. 101, pp. 143–181, 1992.

[2] I.Babuska and W.C.Rheinboldt, "A posteriori error estimators for the finite element method", *Internat. J. Numer. Methods Engrg.*, vol. 12, pp. 1597–1615, 1978.

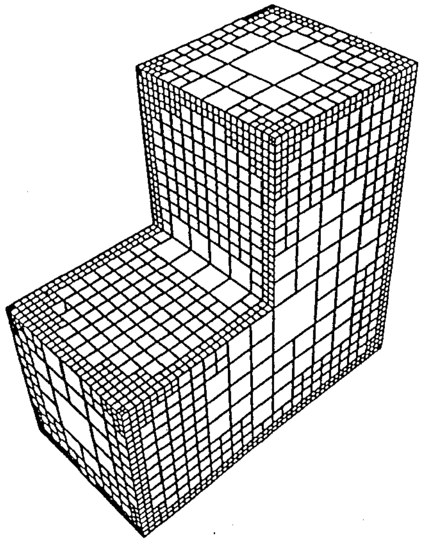
[3] J. Nedelec, "Curved finite element methods for the solution of singular integral equations on surfaces in R^3", *Comp. Meth. in Appl. Mech. and Engrg.*, vol. 8, pp. 61–80, 1976.



uniform		adaptive	
Panels	Cap	Panels	Cap
150	8.204	150	8.204
600	8.262	366	8.254
2,400	8.286	924	8.282
9,600	8.295	2310	8.293

TABLE I

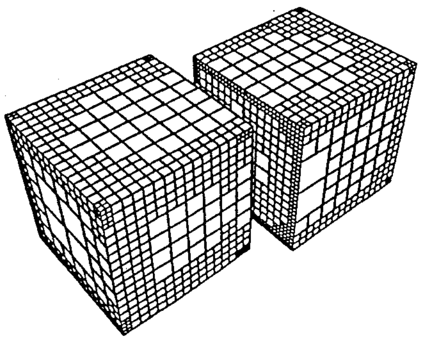
Capacitances for the cube



uniform		adaptive	
Panels	Cap	Panels	Cap
126	12.481	126	12.481
504	12.589	234	12.551
2,016	12.635	528	12.604
8,064	12.653	1,128	12.635
32,256	12.661	4,686	12.657

TABLE II

Capacitances for the L-block



uniform		adaptive	
Panels	self Cap	Panels	self Cap
192	13.871	192	13.871
768	14.161	324	14.033
3,072	14.293	1,008	14.217
12,288	14.349	2,520	14.301
49,152	14.371	5,952	14.341

TABLE III

Capacitances for the two cube structure