Accurate Inductance Extraction with Permeable Materials Using Qualocation

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ABSTRACT

In this paper we present a more accurate method for solving a frequency-dependent inductance of 3-D structures that contains permeable materials. The extracted inductance using the new method is much more accurate than when using the traditional collocation method for no increased cost. Computational results are presented and compared with data to demonstrate the accuracy improvement of the approach.

Keywords: Qualocation, Inductance, Magnetic, permeability.

INTRODUCTION

MEMS devices, such as spiral inductors and magnetic force based actuators, use permeable materials to increase either inductance or magnetic forces. One approach to computing inductance in the presence of non-current carrying magnetic materials is the fictitious magnetic charge method[1,2], in which the material interfaces are replaced with sheets of charge. When the technique is used as a numerical method, the fictitious magnetic charge is computed by enforcing the continuity of normal magnetic field at the permeable material interface at test points. We show that discretizing the resulting numerical system using a collocation method is inaccurate for high μ materials with sharp geometric features and can be replaced by a qualocation method which is much more accurate at no extra cost.

In the next section we briefly describe the fictitious magnetic charge method, and the new technique to solve for the magnetic charge. We also describe how the fields are being computed using the magnetic charge. In section 4 we present some computational experiments. We demonstrate that the new method more accurately predicts the magnetic field for the analytically solvable problem of a permeable ellipsoid in a uniform magnetic field. We also compare the extracted inductance to data for a problem of a coil surrounding a permeable core. We show that new method is much more accurate in predicting the exact inductance. The paper ends with conclusions and acknowledgements.

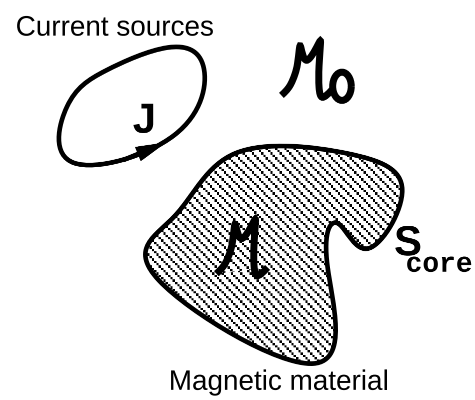

Figure 1: Current sources are outside the magnetic material.

FICTITIOUS MAGNETIC CHARGE METHOD

Integral Formulation

For many problems in MEMS, we can assume that regions which contain magnetic material are separated from current carrying conductors, as shown in Figure 1. For such problems, it is possible to use a fictitious magnetic charge method in which the conductors are divided into filaments over which the current is assumed constant, and the permeable material surface is divided into panels over which the magnetic charge is assumed constant. Then, the filaments are combined into loops. By using these loop and panel basis functions to represent the currents and charges, the resulting system can be solved numerically[2].

The fictitious magnetic charge is computed approximately by solving for the jump in the normal magnetic field at the permeable material interface, as in

$$\rho_m(r)\frac{2\pi(\mu_r+1)}{(\mu_r-1)} = H_C \cdot n_r - \int_S \rho_m(r^{'}) n_r \cdot \nabla \frac{1}{\mid r-r' \mid} dS^{'}_{mag}, \quad (1)$$

where ρ_m is the fictitious surface charge density, r and $r^{'}$ are positions in 3-space, μ_r is the magnetic material's relative permeability, H_C is the magnetic field produced by free space current sources, S is the permeable material's surface, and n_r is the normal to that surface.

Collocation and Galerkin Discretization

The integral equation in (1) can be solved by discretizing the permeable material's surface into panels on which the fictitious charge is assumed constant, and then determining the panel charges by enforcing (1) at test points. This discretization produces the following collocation equation,

$$q_i \frac{2\pi(\mu_r+1)}{(\mu_r-1)} = H_C \cdot n_i - \sum_j q_j \int_{S_j} n_i \cdot \nabla \frac{1}{\mid r_i - r' \mid} dS' \quad (2)$$

where q_j is the fictitious magnetic charge density on the j^{th}panel on the magnetic material surface, n_i is the unit vector normal to the magnetic material surface calculated at the center of panel i, and S_j is the surface of panel j. If the surface is discretized into flat triangles, the integral in (2) can be calculated analytically[3]. Consequently, using the collocation method is efficient.

The galerkin approach can also be used to discretize (1), in which case the panel charges are determined by enforcing (1) in average over a panel. This discretization produces the following galerkin equation,

$$q_i \frac{2\pi(\mu_r+1)}{(\mu_r-1)} = H_C \cdot n_i - \frac{1}{a_i} \int_{S_i} \int_{S_j} q_j n_i \cdot \nabla \frac{1}{\mid r - r' \mid} dS_i dS_j' \quad (3)$$

where a_i is the surface area of panel i.

The double integral in the galerkin method (2) is harder to evaluate than the integral in the collocation method (1), and is typically computed by combining analytic integration with numerical quadrature.

QUALOCATION

If the collocation method with a coarse discretization is used to solve (1), the computed fictitious charges are quite inaccurate when μ is large. Switching to the galerkin method improves accuracy substantially, but the galerkin method is more expensive. Instead, we show that by using a qualocation method[4], which costs no more than collocation, the ficitious charges are also accurately computed.

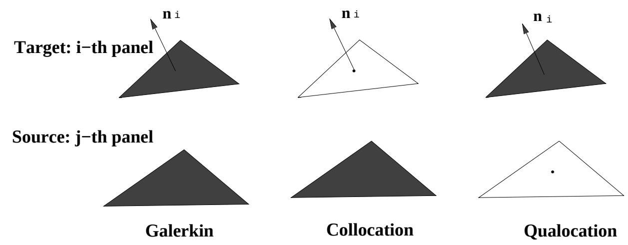

Figure 2: Different discretization configurations

Like the collocation method, the qualocation method can be thought of as an approximation to the galerkin method. In qualocation, however, the panel charge is approximated as a point charge and the integral equation is enforced in average over a panel. This results in the following qualocation equation,

$$q_i \frac{2\pi(\mu_r+1)}{(\mu_r-1)} = H_C \cdot n_i - \frac{a_j}{a_i} \sum_j q_j \int_{S_i} n_i \cdot \nabla \frac{1}{\mid r - r_j' \mid} dS_i' \quad (4)$$

The integral in the qualocation method (4) is equivalent to computing a dipole potential, and can be evaluated analytically[3]. Thus amount of work needed to compute the fictitious magnetic surface charge, and therefore, field and inductance values is almost the same when using the collocation or qualocation method. Figure 2 shows a schematic for the three different discretization methods. The galerkin method is represented by two shaded triangles because the integral in (3) is evaluated over both the source and the target triangles. In the collocation method, the target triangle is not shaded because the integral in (3) is approximated at the target triangle's centroid. However, in the qualocation method, the source triangle is not shaded because the integral over the source triangle is approximated as a point charge at the its centroid.

After computing the magnetic charges, the total magnetic field can be calculated from

$$H(r)_{Total} = H_C - \sum_j q_j \int_S \nabla \frac{1}{\mid r - r' \mid} dS_j'. \quad (5)$$

RESULTS

To demonstrate the accuracy of the new method, we examined the analytically solvable problem of an ellipsoid of permeable material in a uniform vertical magnetic field, as shown in Figure 3. The magnetic charge was solved for using (4), then flux density was calculated using (5). To compare with the collocation method, we used (2), instead of (4), to solve for the magnetic charge.

As shown in Figure 4, the average flux density over the median cross section of the ellipsoid, shown in Figure 3,

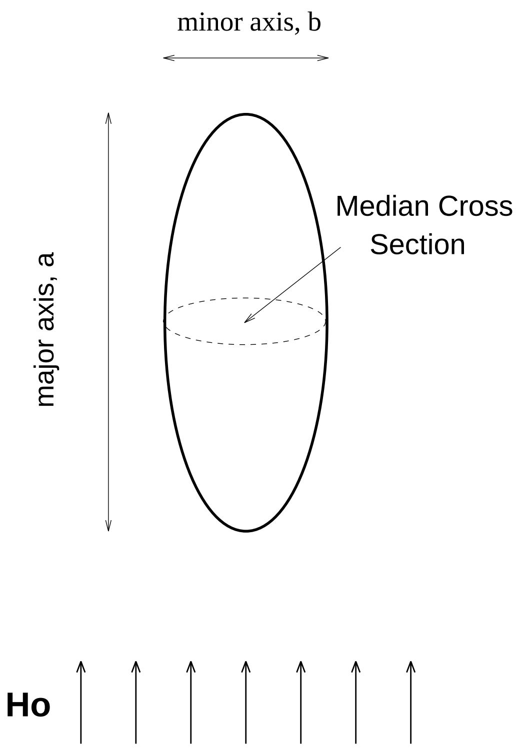

Figure 3: Ellipsoid with $\mu_r = 1000$ in Ho, b=1 m.

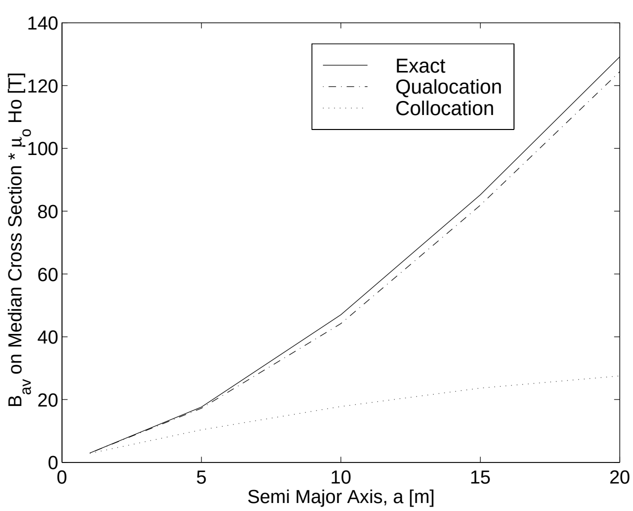

Figure 4: Average flux density over the ellipsoid's median cross section, shown in figure 3, for both collocation and qualocation.

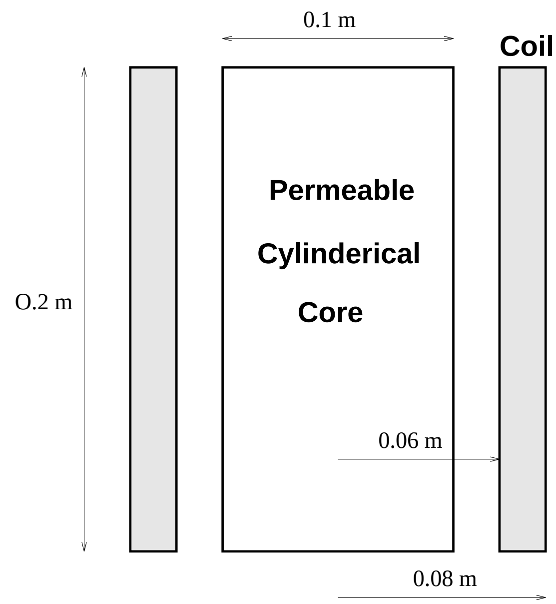

Figure 5: Permeable cylinder and excitation coil.

computed using the qualocation method matches the analytic solution much more accurately than when using the collocation method, with larger difference associated with thinner ellipsoids.

To show the qualocation method improves the extracted inductance accuracy, we considered a coil of wire around a permeable cylinder, as shown in Figure 5. The current density in the coil was constant. The inductance was calculated as follows,

$$Inductance = \frac{\int_S B(r) \cdot dS}{I}, \tag{6}$$

where I is the total current flowing in the coil, S is the surface of the coil median surface, and B is the magnetic flux density evaluated using (5). H_C in (5) is determined from the current density using,

$$H_C(r) = \nabla \times \int_v \frac{J(r')dv'}{\mid r - r' \mid}, \tag{7}$$

where J is the current density, and v is the volume of coil.

The inductance was computed using both collocation and qualocation, and the results compared with data [5]. As shown in figure 6, qualocation is much more accurate. The inaccuracy of the collocation approach is most clearly demonstrated by examining a coil of wire around a long permeable cylinder, shown in Figure 7. Figure 8 shows that the collocation underestimates the magnetic charges on top of the permeable cylinder.

CONCLUSIONS AND ACKNOWLEDGEMENTS

An enhanced method for solving a frequency-dependent inductance of 3-D structures that contains permeable ma-

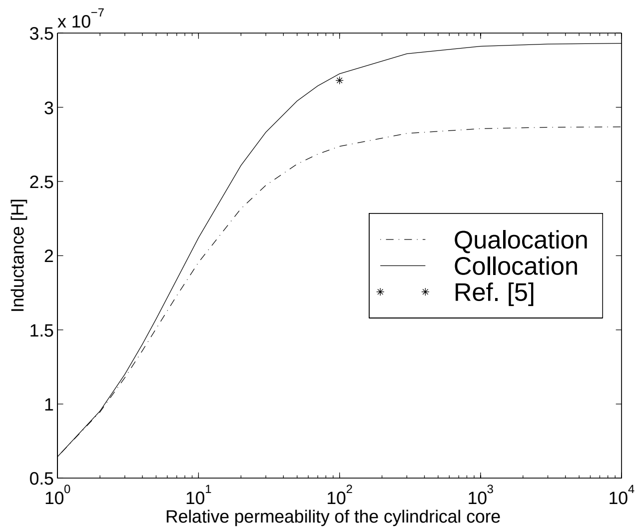

Figure 6: Comparison between extracted inductance using collocation and qualocation. X notes data from [5].

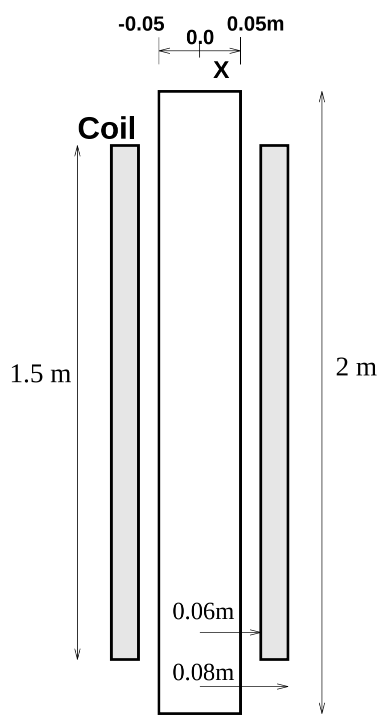

Figure 7: Coil surrounding $\mu_r = 1000$ cylinder.

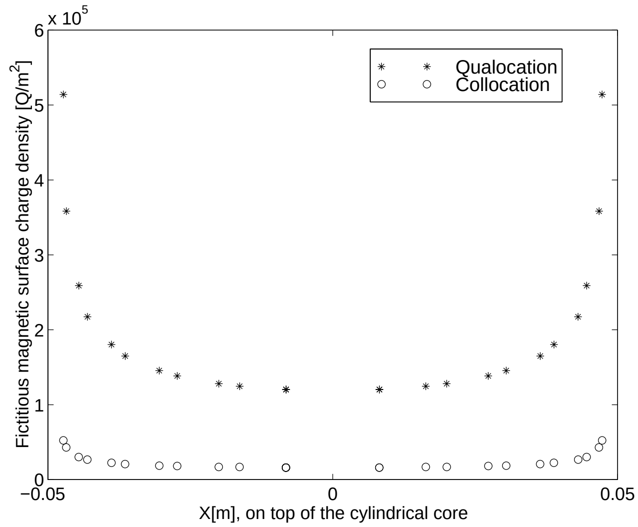

Figure 8: Fictitious magnetic surface charge on cylinder top.

terials has been presented. The new method shows is no more expensive in than collocation and is up to an order of magnitude more accurate for highly permeable structures with big aspect ratios or with edges. Computational results were presented and compared with experimental data to demonstrate the accuracy improvement of the approach.

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