A Boundary-Element approach to Transient Simulation of Three-dimensional Integrated Circuit Interconnect

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Abstract

It has been recently suggested that sufficiently accurate integrated circuit cross-talk simulations can be performed by computing the time evolution of electric fields both inside and outside three-dimensional integrated circuit conductors via a finite-difference discretization of Laplace's equation. In this paper the same calculation is performed, but the volume mesh associated with finite-difference methods is avoided through the use of a boundary-element method in which only conductor surfaces are discretized. Two boundary-element approaches are investigated, and it is shown that the straight-forward approach leads to unacceptable discretization errors, and a less intuitive second approach yields good results even with coarse surface meshes. Finally, numerical experiments demonstrating the effectiveness of the second approach in calculating cross-talk are presented.

1 Introduction

A simplified approach to analyzing the parasitic coupling between nearby lines of interconnect on an integrated circuit, referred to as cross-talk, is to compute a lumped resistor-capacitor model for the interconnect structure. Such an approach provides a certain amount of insight, but doesn't model the distributed effects accurately enough to make aggressive design decisions. Recently it was suggested that sufficiently accurate cross-talk simulations could be performed by computing the time evolution of the electric fields both inside and outside the integrated circuit conductors via a finite-difference discretization of Laplace's equation[4]. In that approach, the simplified equations were derived by assuming the magnetic fields and volume charges were negligible.

In this paper, we perform this same calculation but avoid the volume mesh associated finite-difference methods. To see why this is feasible, consider that the absence of volume charge and magnetic fields imply that currents which flow through conductors only serve to increase or decrease conductor surface charge with time. This suggests that a boundary-element method, in which only conductor surface quantities are computed, can capture the transient phenomenon. In this paper, two boundary-element approaches are investigated, and it is shown that the most straightforward approach leads to unacceptable discretization errors, and a less intuitive second approach yields good results even with coarse surface meshes. Finally, numerical experiments demonstrating the effectiveness of the second approach in calculating cross-talk are presented.

In the next section, our assumptions about the cross-talk problem are given, and several basic results derived. In section 3, we derive a source formulation for the transient interconnect problem, and demonstrate the difficulties with discretizations of that approach. A Green's theorem based formulation is derived in section 4, and its advantages compared to the source formulation are made clear. In section 5, we present some application results. Finally, we give our conclusions and acknowledgements in Section 6

2 Basic Assumptions and Consequences

For homogeneous dielectric media, the electric field, E satisfies

$$\nabla \cdot E = \frac{\rho}{\epsilon}, \tag{1}$$

where ρ is the volume charge density and ϵ is the dielectric permitivity. Inside a conductor, the current density, J, is given by $J = \sigma E$ where σ is the conductivity of the material. Conservation of charge implies that

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t} \tag{2}$$

and substituting for J in terms of the electric field yeilds

$$\nabla \cdot E = -\frac{1}{\sigma}\frac{\partial \rho}{\partial t}. \tag{3}$$

Combining (1) and (3) leads to an equation for the time evolution of the volume charge inside a conductor, as in

$$\frac{\partial \rho}{\partial t} = -\frac{\rho}{\tau} \tag{4}$$

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where $\tau = \frac{\epsilon}{\sigma}$ is the dielectric relaxation time, and is of the order of hundreds of femtoseconds. Note that at such time scales, the assumed constitutive relations between J and E are not likely to still apply, so (refeq:qdecay) should only be used as an indicator of longer time behavior.

From (4), it follows that any initial charge in the interior of a conductor must rapidly decay, and this volume charge *never* increases, regardless of what happens on the periphery of the conductor. For this reason, volume charge inside the conductor can *always* be neglected. The computationally important implication of zero conductor volume charge is that all currents flow through the conductor to the conductor surface, where they produce a build-up of surface charge. Note that this surface charge may be "bled off" by external circuitry at points where contact is made to the conductor.

It is generally assumed that for integrated circuit interconnect, length scales are such that magnetic effects can be ignored, though this may not continue to be the case as integrated circuit speeds increase. This assumption implies that the electric field, E, is given by $E = \nabla\psi$, where ψ is a scalar potential. Also, there is zero volume charge in the free space outside the conductor, and as shown above, zero volume charge inside the conductor. Together, these several observations imply that (1) can be simplified to

$$\nabla^2\psi = 0 \tag{5}$$

everywhere except on the conductor surface. Therefore, ψ can be related to the conductor surface charge, ρ_s, through the superposition integral

$$\psi(x) = \int_S \frac{\rho_s(x')}{4\pi\epsilon\|x - x'\|} da' \tag{6}$$

where S is the union of all the conductor surfaces, and $\|x - x'\|$ is the Euclidean distance between x and x'.

As conductor surface charge is now the only charge in the problem, we have from conservation of charge that at any point x on the conductor surface which is not an external contact point,

$$-\frac{\partial\rho_s(x)}{\partial t} = J_{normal}(x) = -\sigma\frac{\partial\psi(x)}{\partial n}, \tag{7}$$

and on any point x on the conductor surface which is a contact point

$$\begin{aligned} -\frac{\partial\rho_s(x)}{\partial t} &= J_{normal}(x) - J_{external}(x) \\ &= -\sigma\frac{\partial\psi(x)}{\partial n} - J_{external}(x) \end{aligned} \tag{8}$$

where $J_{normal}(x)$ is the current density along the inward directed normal to the conductor surface, $J_{external}(x)$ is the current density supplied to the conductor through the contact, ρ_s is the conductor surface charge, and $\frac{\partial\psi(x)}{\partial n}$ is the spatial derivative of ψ along the inward directed normal to the conductor surface.

3 Source Formulation

In this section, a physically simple boundary integral formulation for the transient interconnect problem is presented, along with a discretization procedure. An example is given to demonstrate that the solutions obtained by discretizing this simple formulation have unacceptable errors. In the next section, a less intuitive formulation is given which allieviates these difficulties.

3.1 Integral Formulation

A simple boundary formulation for the transient interconnect problem can be derived by eliminating the normal derivative of the potential from (7) and (8) by differentiating (6) and substituting. For non-contact points on the conductor surface this yeilds

$$-\frac{\partial\rho_s(x)}{\partial t} == -\sigma\int_S \frac{\partial}{\partial n}\frac{\rho_s(x')}{4\pi\epsilon\|x - x'\|} da', \tag{9}$$

and for external contact points,

$$-\frac{\partial\rho_s(x)}{\partial t} = -\sigma\int_S \frac{\partial}{\partial n}\frac{\rho_s(x')}{4\pi\epsilon\|x - x'\|} da' - J_{external}(x). \tag{10}$$

3.2 Charge Discretization

To numerically solve (10) for ρ_s at non-contact conductor surfaces, and both $J_{external}$ and ρ_s at contact surfaces, the surfaces of all the conductors are broken into small panels or tiles. It is then assumed that on each panel l, there is a constant surface charge q_l. In addition, it is assumed that for each panel of a contact surface, l_c, there is a constant current density J_c.

If there are n total panels, m of which belong to contact surfaces, then there are n unknown panel charges, and m unknown contact panel currents, for a total of $m + n$ unknowns. To generate a system of $n + m$ equations from which these $n + m$ unknowns can be computed, a collocation scheme is used[2][5]. That is, (9) is enforced at the center point in each of $n - m$ non-contact panels and (10) is enforced at the center point of each of the m contact panels. The result is a linear system of the form

$$4\pi\tau\frac{\partial}{\partial t}q(t) = -Dq(t) + AJ_c(t). \tag{11}$$

where $q(t) \in \Re^n$ is the vector of time-varying panel charges, $J_c(t) \in \Re^m$ is the vector of time-varying contact panel currents, $A \in \Re^{n\times m}$ is a matrix of nearly all zeros, except $A_{l,l-(n-m)} = 1$ if l is a contact panel, and $D \in \Re^{n\times n}$ is given by

$$D_{kl} = \frac{\partial}{\partial n'}\int_{panel_l}\frac{1}{\|x' - x_k\|} da'; \tag{12}$$

where x_k is the center of panel k. Note that in defining A, it is assumed the elements of J_c are appropriately normalized.

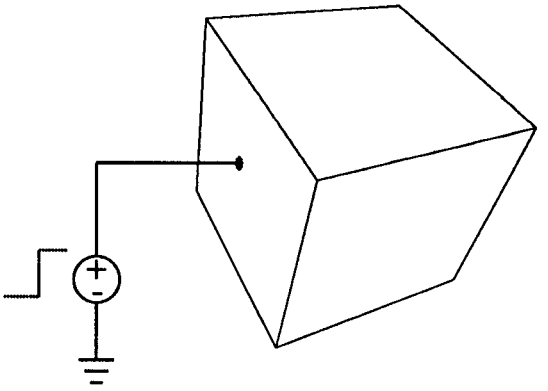

Figure 1: Conductive cube with one side driven by a voltage step.

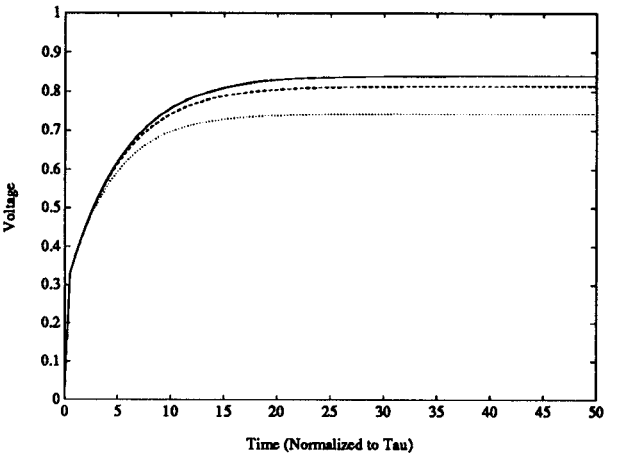

Figure 2: Potential computed at the observation point using three different uniform surface discretizations of the source formulation. The dotted line is the result of using 54 panels, the dashed line the result of using 486 panels, and the solid line is the result of using 1944 panels.

Given that charge density is assumed constant on each panel, from (6) it follows that the m contact panel potentials, which are assumed constant over the panel, are related to the n panel charges by

$$\Psi_c = Pq \tag{13}$$

where $\Psi_c \in \Re^m$ is the vector of contact panel potentials and $P \in \Re^{m \times n}$ is given by

$$P_{kl} = \frac{1}{a_l} \int_{contact\ panel_l} \frac{1}{\|x' - x_k\|} da' \tag{14}$$

where a_l is the area of the l^{th} panel, and x_k is the center of the k^{th} panel.

3.3 Numerical Difficulties

The combination of (11) and (13) yeilds a differential-algebraic system of $n+m$ equations in $n+m$ unknowns. It is straight-forward to solve this system numerically for the time evolution of the panel charges, and to use (6) to compute potentials at any conductor surface point of interest. Unfortunately, the results from such a calculation contain spatial discretization errors that preclude the scheme's use as part of a circuit simulation program.

This numerical difficulty can best be demonstrated by examining a simple example, such as the conductive cube shown in Figure (1). One face of the cube is assumed to be driven by an ideal voltage source which makes a unit step transition, and therefore this driven cube face is an isopotential surface whose potential changes, at time zero, from zero to one volt. The correct behavior of the potentials at points on non-driven cube surfaces is obvious, they should change, over time, from zero to one volt.

Figure (2) is a plot of the time behavoir of the potential at an observation point on the cube face opposite the driven face, for three sucessively finer surface discretizations. As even the plot based on discretizing the surface into 1944 panels shows, the discretization error is such that the computed equilibrium, or final value, for the potential is only about 0.85 volts. Such an error is clearly unacceptable if the intention is to use the calculation procedure as part of a circuit simulator.

4 Green's Formulation

The nonphysical nature of the discretization error in the source formulation stems primarily from the fact that charge is discretized, rather than potential. This can be remedied by switching to a formulation based on Green's identity. Using this second formulation allows the potential to be discretized directly, and insures that isopotential equilibrium behavior is retained *regardless* of the spatial discretization. In the three subsections below we derive this Green's identity based formulation, describe the numerical discretization procedure, and compare computed results with those produced using the source formulation to demonstrate the Green's formulation's advantages.

4.1 Formulation Derivation

As ψ satisfies (5), it is easily derived from Green's theorem that[3]

$$\frac{\psi(x)}{\epsilon} = \int_{S_i} \psi(x') \frac{\partial}{\partial n'} \frac{1}{4\pi\epsilon\|x-x'\|} da' - \int_{S_i} \frac{\partial \psi(x')}{\partial n'} \frac{1}{4\pi\epsilon\|x-x'\|} da', \tag{15}$$

where S_i is the closed surface of the i^{th} conductor, and x is some point on S_i. It also follows from Green's

theorem that if x is outside conductor i, then

$$\int_{S_i} \psi(x') \frac{\partial}{\partial n'} \frac{1}{4\pi\epsilon\|x-x'\|} da' \qquad (16)$$
$$- \int_{S_i} \frac{\partial \psi(x')}{\partial n'} \frac{1}{4\pi\epsilon\|x-x'\|} da' = 0.$$

Summing appropriately (15) or (16) over all the conductor surfaces yields

$$\frac{\psi(x)}{\epsilon} = \int_S \psi(x') \frac{\partial}{\partial n'} \frac{1}{4\pi\epsilon\|x-x'\|} da' \qquad (17)$$
$$- \int_S \frac{\partial \psi(x')}{\partial n'} \frac{1}{4\pi\epsilon\|x-x'\|} da'.$$

where S is the union of the conductor surfaces S_i, and x is some point in S.

Decomposing the second integral in (17) into contact and non-contact surfaces, and then substituting using (7) and (8) leads to

$$\frac{\psi(x)}{\epsilon} = \int_S \psi(x') \frac{\partial}{\partial n'} \frac{1}{4\pi\epsilon\|x-x'\|} da' \qquad (18)$$
$$- \int_{S_{noncontact}} \frac{1}{\sigma} \frac{\partial \rho(x')}{\partial t} \frac{1}{4\pi\epsilon\|x-x'\|} da'$$
$$- \int_{S_{contact}} \frac{1}{\sigma} \frac{\partial \rho(x')}{\partial t} \frac{1}{4\pi\epsilon\|x-x'\|} da'$$
$$+ \int_{S_{contact}} \frac{1}{\sigma} J_{external}(x') \frac{1}{4\pi\epsilon\|x-x'\|} da'.$$

Two of the integrals in (18) can be eliminated by noticing that the time-derivative of (6) yields

$$\frac{\partial \psi(x)}{\partial t} = \int_S \frac{\partial \rho_s(x')}{\partial t} \frac{1}{4\pi\epsilon\|x-x'\|} da'. \qquad (19)$$

Combining (18) with (19), and then substantially reorganizing, produces

$$4\pi\tau \frac{\partial \psi(x)}{\partial t} = -4\pi\psi(x) + \int_S \psi(x') \frac{\partial}{\partial n'} \frac{1}{\|x-x'\|} da' \qquad (20)$$
$$+ \frac{1}{\sigma} \int_{S_{contact}} \frac{J_{external}(x')}{\|x-x'\|} da'$$

where again note that

$$J_{external}(x) = \frac{\partial \rho_s(x)}{\partial t} - \sigma \frac{\partial \psi(x)}{\partial n'} \qquad (21)$$

is, not surprisingly, the sum of displacement and resistive current at the contact point x.

Although perhaps tedious to derive, (20) has the advantage of including as unknowns only the variables of interest, that is, only the surface potentials and the contact surface currents.

4.2 Discretization

To numerically solve (20) for ψ at non-contact conductor surfaces, and for J at contact surfaces (as defined in (21)), the surfaces of all the conductors are broken into small panels or tiles. It is then assumed that on each panel l, there is a constant surface potential Ψ_l. In addition, it is assumed that for each panel of a contact surface, l_c, there is a constant current density J_c.

If there are n total panels, m of which belong to contact surfaces for which the potential is known a-priori, the vector of constant panel potentials, $\Psi \in \Re^n$, has $n-m$ unknown entries. As none of the entries of the vector of m constant contact panel currents, $J_c \in \Re^m$, is known a-proiri, a total of n unknowns must be determined. To generate a system of n equations from which the unknown elements from the vectors Ψ and J_c can be computed, as with the source formulation a collocation scheme is used[2][5]. That is, (20) is enforced at the center point in each of n panels. The result is a dense linear system of the form

$$4\pi\tau \frac{\partial}{\partial t} \Psi = (D - 4\pi)\Psi + PJ \qquad (22)$$

where $D \in \Re^{n \times n}$ whose elements are

$$D_{kl} = \int_{panel_l} \frac{\partial}{\partial n'} \frac{1}{\|x' - x_k\|} da'; \qquad (23)$$

and where $P \in \Re^{n \times m}$ whose elements are

$$P_{kl} = \frac{1}{a_l} \int_{contact\ panel_l} \frac{1}{\|x' - x_k\|} da' \qquad (24)$$

where x_k is the center of the k^{th} panel. Note here that the elements of J here are normalized by σ.

4.3 Comparison to the Source Formulation

The Green's identity based formulation has several advantages over the source formulation. The discretization of the source formulation produces a system of $n+m$ equations in $n+m$ unknowns, and the discretization of the Green's formulation produces an system of only n equations in n unknowns. In addition, all the surface potentials are calculated in the case of the discretized Green's formulation, but in the case of the discretized source formulation the surface potentials at points of interest must be derived from integrals of computed surface charges.

To show the superior accuracy in the surface potential produced by the discretized Green's formulation, again consider the simple conductive cube example shown in Figure (1). Figure (3) is a plot of the time behavior of the potential at an observation point on the cube face opposite the driven face, for three sucessively finer surface discretizations. As comparing the plots in Figures (2) and (3) make clear, the discretized Green's formulation converges with panel refinement

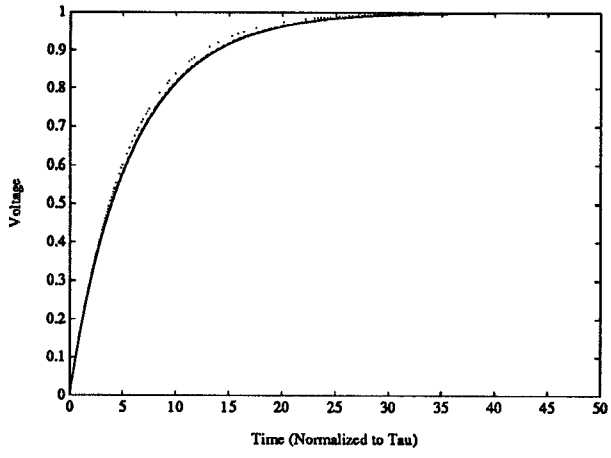

Figure 3: Potential computed at observation point in Figure 1, using three different uniform surface discretizations of the Green's formulation. The dotted line is the result of using 54 panels, the dashed line the result of using 486 panels, and the solid line is the result of using 1944 panels.

much more rapidly than the discretized source formulation, and in addition, the discretized Green's formulation computes the correct equilibrium, or final value, of the potential *regardless of the coarseness of the discretization.*

5 Application Experiments

The most obvious application for the above technique is determining how much changes in voltages on a given conductor are capacitively transmitted to nearby conductors. In particular, we consider the scenario depicted in Figure (4), in which a voltage step is applied to the near end of one conductor, and its effect is monitored at the far end of a parallel conductor which has its near end grounded. The intent is to model realistic interconnect, in which typically one end of every conductor is driven and the other end is connected to a high-impedence input. Clearly it is difficult to approximate how much the voltage will rise at the far end of the grounded conductor.

To compute this voltage rise on the far end of the grounded conductor as a function of the length of the parallel pair, we used the discretized Green's formulation given in (22) combined with a fixed-timestep backward-Euler algorithm, and used 200 fixed timesteps. The resulting matrix problems were solved using one L-U factorization, and 200 forward-elimination/backward-substitutions. To examine the length effects, the conductors were each assumed to be of unit square cross-section and a unit distance apart. Transient solutions were computed for conductors of 2, 4, 6, and 8 units. The surface discretizations for the length 2 and 8 computations are given in Figures (5)

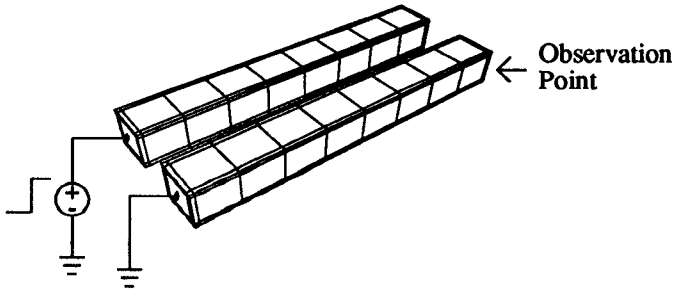

Figure 4: Cross-Talk Simulation Scenario

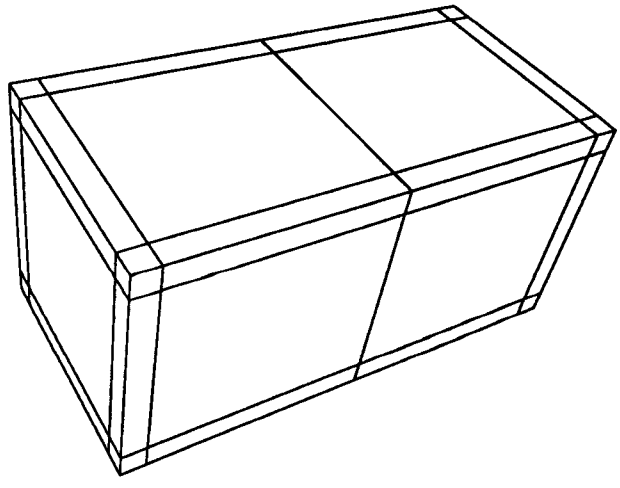

Figure 5: Two-unit length discretization

and (4) respectively, and the transient behaviors for several lengths are plotted in Figure (6).

Note that the time-axis in all these plots is normalized to $\tau = \frac{\epsilon}{\sigma}$. This makes a point that was not initially obvious to the authors, that the *peak magnitude* of the cross-talk pulse is uneffected by changes in either the conductivity, σ, or the dielectric permittivity ϵ. Though, of course, the period of the pulse is directly proportional to $\tau = \frac{\epsilon}{\sigma}$.

Finally, the key point of this work is demonstrated in Table 1, in which the number of panels, or unknowns, and the CPU time required for the transient calculation on an IBM RS6000 model 540 is presented. Note that the most expensive calculation requires less than half a minute, which compares very favorably to computation times required by finite-difference based techniques[4]

length	unknowns	CPU Time
2	132	4.7s
4	180	8.9s
6	228	14.2s
8	276	30.8s

Table 1: CPU times for Transient Calculation

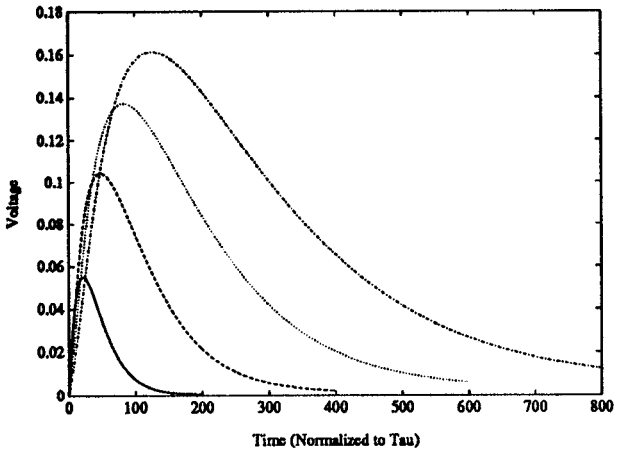

Figure 6: Cross-Talk waveforms for several interconnect lengths. The solid line is for a 2-unit long interconnect, the dashed line for a 4-unit long interconnect, the dotted line for a 6-unit long interconnect, and the dash-dotted line for a 8-unit long interconnect.

6 Conclusions and Acknowledgements

In this paper it is demonstrated that boundary-element techniques can be used to perform very efficient transient simulation of three-dimensional interconnect structures, fast enough to easily be included in a circuit simulator. It was first shown that reasonable discretizations of the most obvious integral formulation do not produce results of acceptable accuracy, and a less intuitive formulation was derived and shown to insure reasonable results even for coarse discretizations.

Note that for the experiments presented in this paper, the differential-algebraic system in (22) was solved using a simple backward-Euler scheme. However, a variety of techniques such as backward-difference methods[1] or moment-matching algorithms[4] can also be used. Also, although the system in (22) is dense, the formulation has advantages over using finite-difference or finite-element techniques applied to solving (5) directly. Since only the two-dimensional conductor surfaces are discretized, rather than the three-dimensional volume containing the problem, as would be the case with finite-difference methods, the discretization is easier to generate and the resulting system contains many fewer unknowns. Unfortunately, the resulting matrix problem is dense, and therefore the comparison to finite-difference methods is not clear.

Note that the direct factorization algorithm used in the numerical experiments presented above limits the size of problems which can be analyzed to relatively simple, but still interesting, structures. In addition, homogeneous dielectric media was assumed, and this may not be realistic. To address these short-comings, we are investigating using iterative techniques to perform the matrix solution, as well as trying to improve the efficency of the matrix coefficient calculation using multipole algorithms. Also, we are examining how to include dielectric interfaces in our formulation.

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